

ODE PhD Entrance Exam Outline

Topics:

1. Linear homogeneous systems with constant coefficients:
The general solution, the fundamental matrix, the matrix exponential, phase diagrams in 2-dimensions, stability of equilibrium solutions.
2. General linear systems:
The fundamental matrix, nonhomogeneous systems.
3. Linearization about critical points of nonlinear autonomous systems
Phase plane portraits in 2-dimensions.
4. Theory of Existence, uniqueness, and continuity.
5. Stability analysis by perturbation (or linearization) and Liapunov method.
6. Existence and stability of periodic solutions in 2-dimensions:
Poincare-Bendixson Theorem. Floquet theory.
7. Bifurcation analysis and center, stable, unstable manifolds:
Saddle-node, transcritical, pitchfork, Hopf bifurcations.

Reference Books:

1. Nonlinear Ordinary Differential Equations, Third Edition, by D. W. Jordan and P. Smith
Oxford Applied Math and Computing Science Series (1999)
Chapters 1, 2, 4, 8, 9, 10, 11, 12, and Appendix A.
2. Qualitative Theory of Ordinary Differential Equations by F. Brauer and J. Nohel
W. B. Benjamin, Inc, New York, (1969)
Chapters 1, 2, 3, 4, 5, 6
3. Differential Equations and Dynamical Systems by Lawrence Perko.
Springer-Verlag
Chapters 1, 2 (except 2.13), 3 (3.1 to 3.9), 4 (4.1 to 4.4).

Remarks:

Students could use one of the reference books as main resource for preparation and use other books for some specific topics if necessary. Perko's treatment of Liapunov method seems weaker than other two reference books. Jordan and Smith's treatment of center, stable, unstable manifold is weak. There is no treatment of topic #7 in Brauer and Nohel.