SENR/NRPy: A Next-Generation, Dynamical Reference Metric Numerical Relativity Code

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in collaboration with

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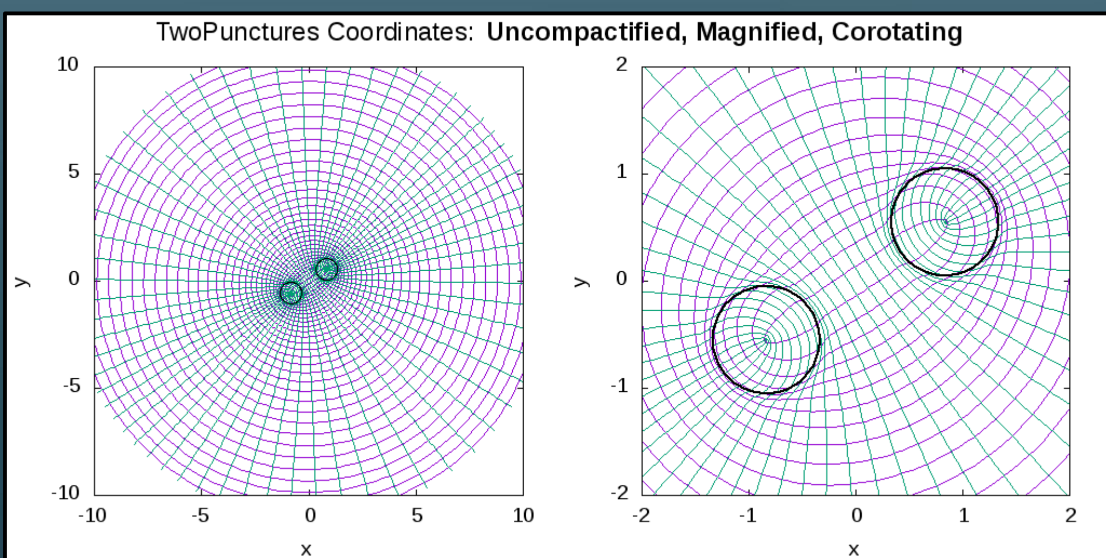

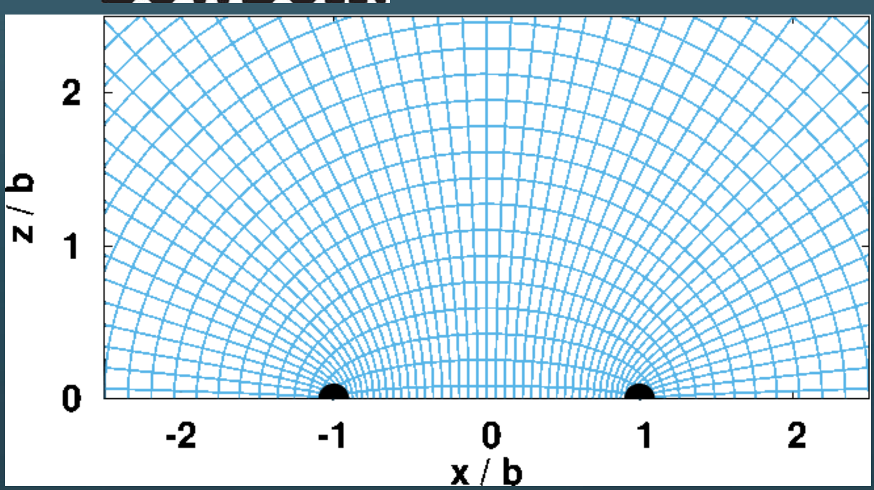

SENR/NRPy: Code Overview

- NRPy: Python+sympy code generation for NR
  - Similar to Kranc, but with no Mathematica!
    - Equations *at your fingertips*, even on HPC systems!
    - Input: Einstein notation + simple syntax Python code
    - Output: Efficient, compiler-vectorizable C code (AVX)
- SENR: Simple, Efficient Numerical Relativity code
  - Contains NRPy wrappers, diagnostics, MoL, BCs for solving BSSN equations in arbitrary coord systems
    - Log-Spherical Polar, Cylindrical, Cartesian, Bispherical-like

https://tinyurl.com/senrcode

SENR/NRPy: Motivation

- SENR/NRPy = Simple, efficient, open (BSD-licensed, Python-based) infrastructure for numerical relativity codes and code generation
- Goal: When solving problem, choose the best coordinate grid for the task!
  - Black hole, neutron star: Log-Spherical polar coords
  - Compact binary: Dynamical, Bispherical-like coords
- Better coordinate grids = Giant efficiency gain over AMR!
  - At least ~160x decrease in # of gridpoints → use desktop for BHB
  - Single grid patch = ~25x better scalability than AMR!

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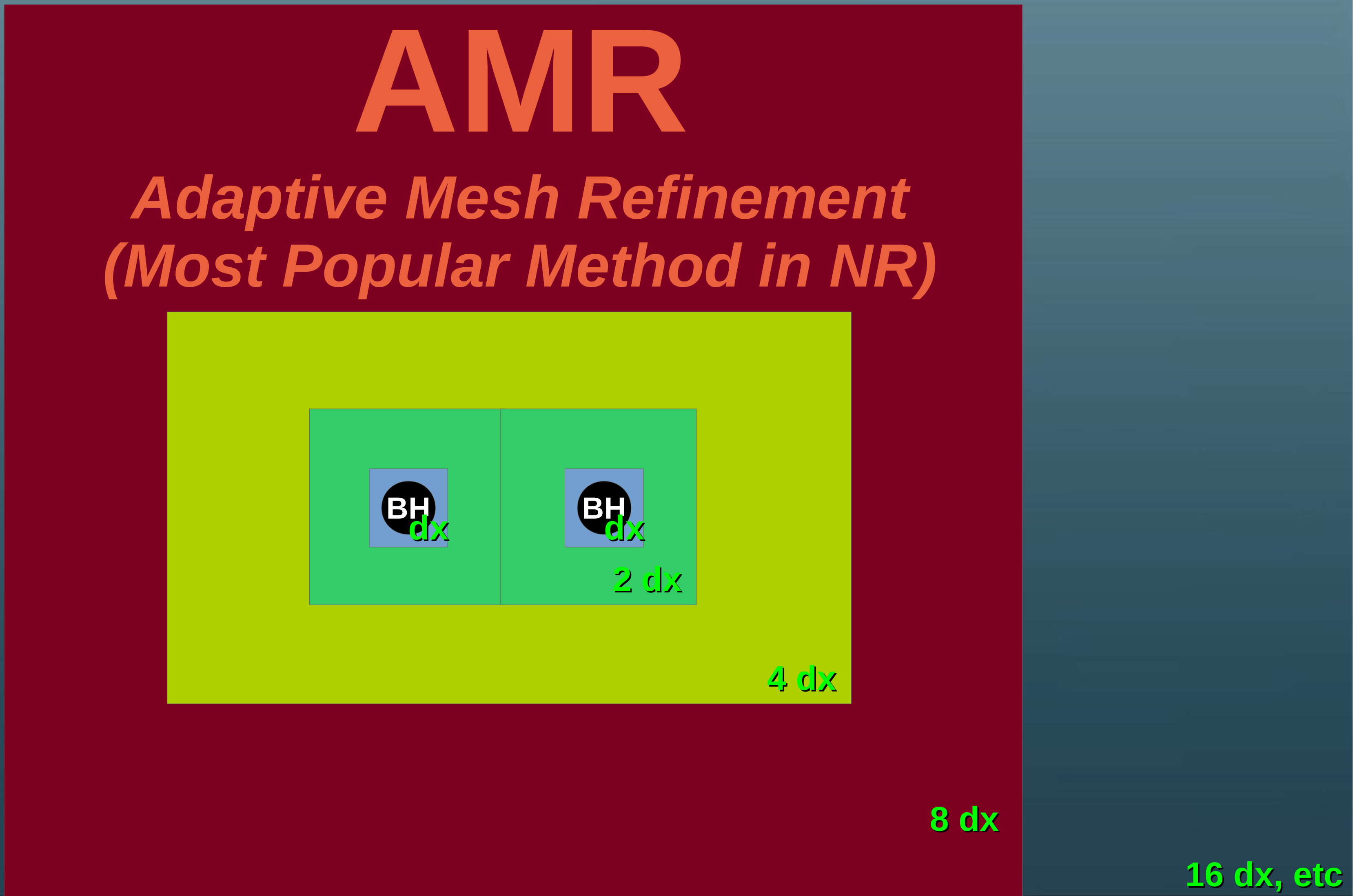
Enormous Inefficiencies Exist in Numerical Relativity (NR) Simulations



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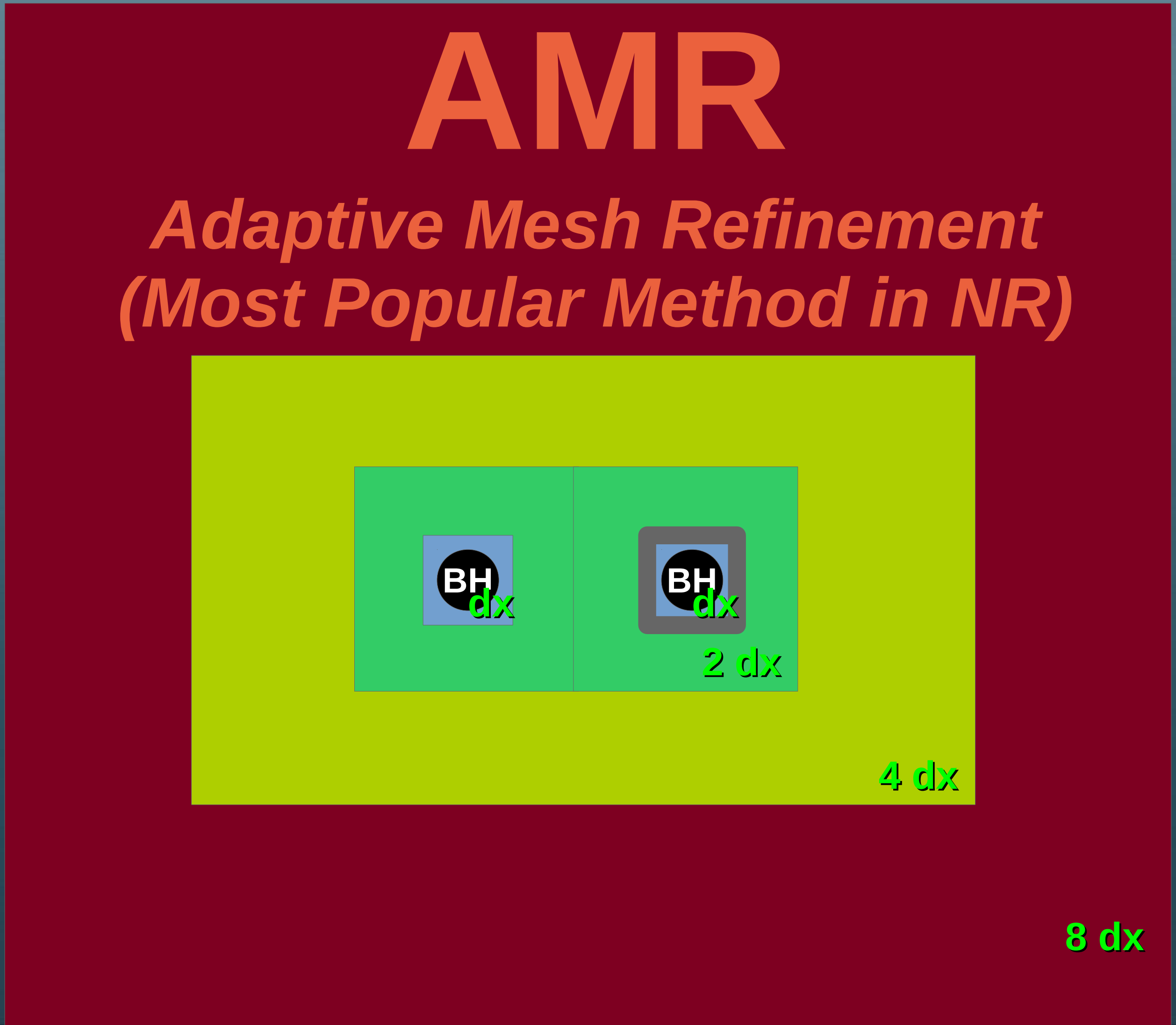

16 dx, etc

Enormous Inefficiencies Exist in Numerical Relativity (NR) Simulations

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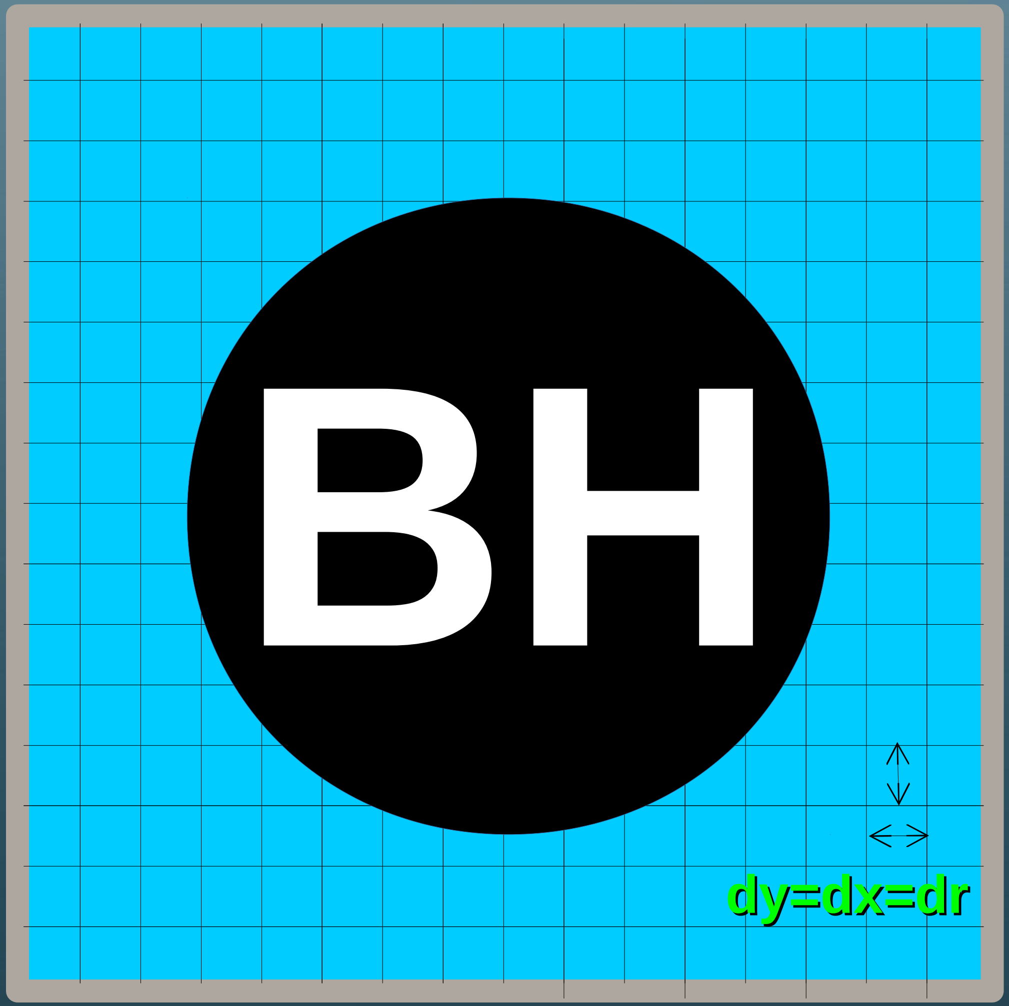
Near-Spherical Object

- **Highest res needed in radial dirn, need ~1/3 points in angular directions Cost: Nr*Ntheta*Nphi ~ 1/9 Nr^3**
- **Cartesian grid: need dx=dy=dz=dr. Cost: Nx*Ny*Nz ~ Nr^3**
- **So far, spherical polar grid ~9x more efficient than Cartesian**



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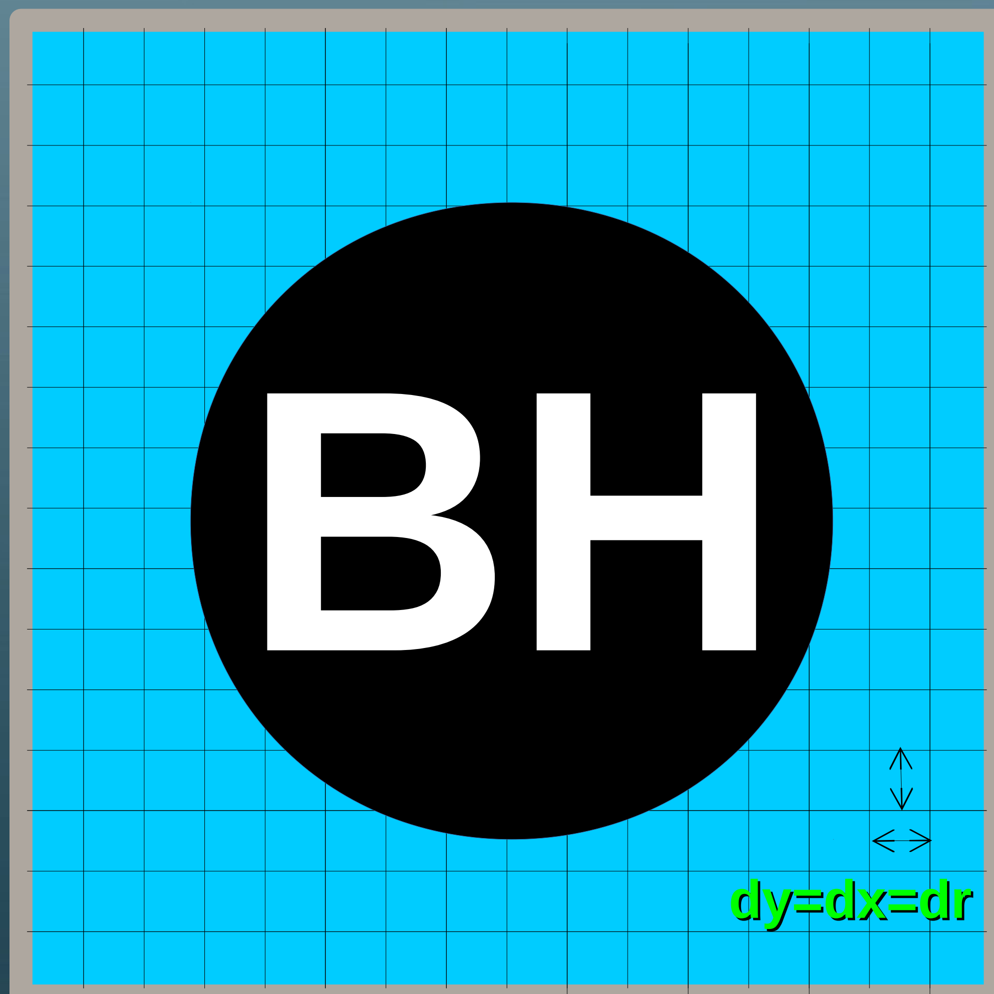

What about dr along diagonal?

- Cube diagonal = √3*sidelength → to get dr resolution in all directions, need to reduce dx,dy,dz by √3
- Since cost in memory ~$1/dx^3$, "fitting the round peg in a square hole" increases cost by another factor of $(\sqrt{3})^3$~5.2x!

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Inefficiencies so far: ~47x

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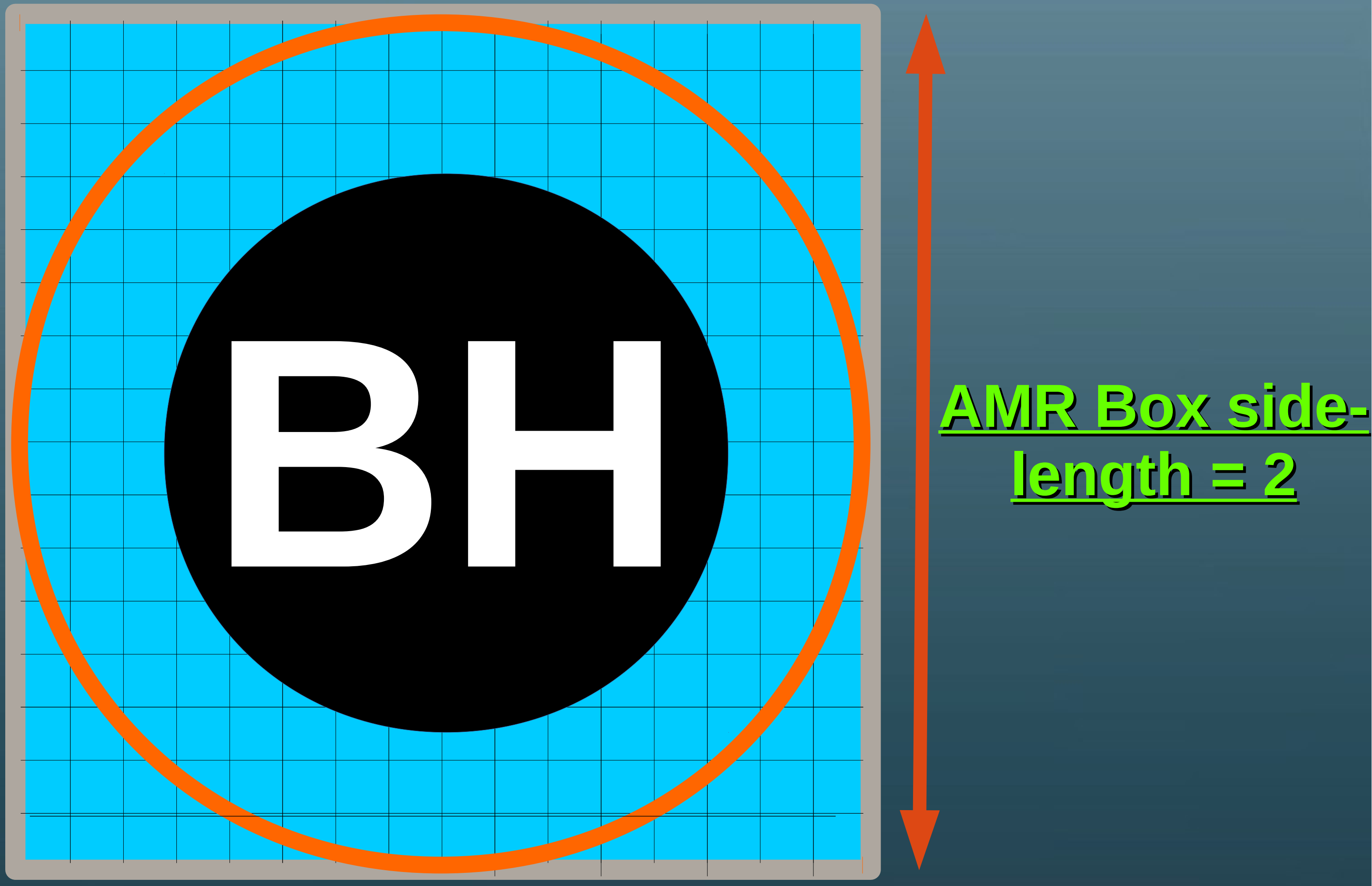
AMR Box Boundary is a Cube...

- ... but fields fall off radially!
- → region outside orange circle is over-resolved by 2x
- Total volume of over-resolved region = 8-4/3 pi ~ 3.8 = about half the cube!
- Bispherical coordinate system: Gain another ~1.7x



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AMR Box side-length = 2

Inefficiencies so far: ~80x

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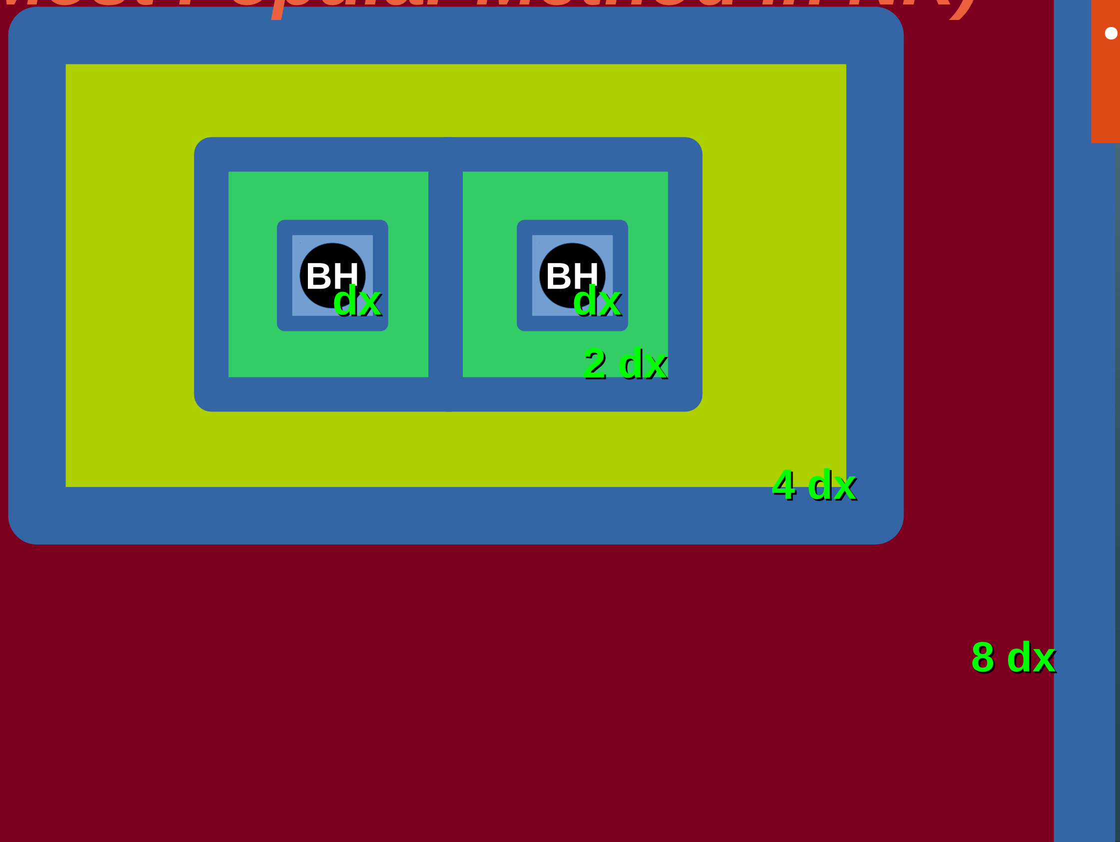
AMR

Adaptive Mesh Refinement (Most Popular Method in NR)



AMR

- Information must be interpolated across refinement boundaries.
- Interpolation → grids must overlap
- Overlap regions (grey) can take up 50% of overall computational domain!

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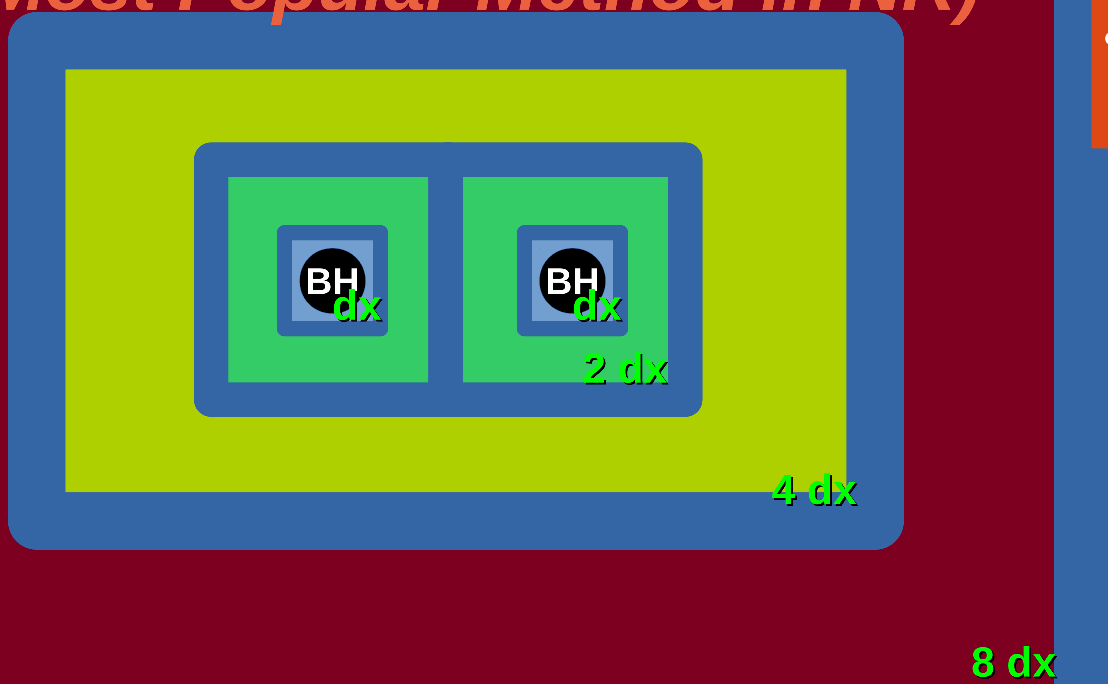

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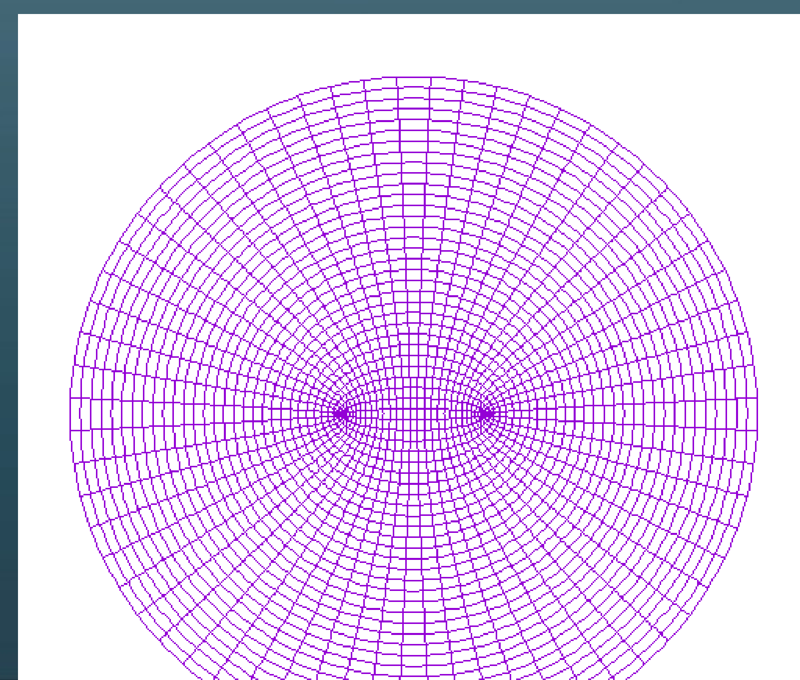

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High-order finite difference with AMR

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Inefficiency Measurement

- Single black hole
  - Moderate spin:
    - a/M = 0.5
- Set up AMR (Carpet) grid, measure H constraint violation
- Adjust SENR grids:
  - H_SENR < H_AMR *at all points*

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AMR Grid:
- <u>10 GB</u>

SENR's Log-Spherical Polar Grid:
- <u>40 MB</u>
- (un-optimized grid structure, another 4-10x drop possible)

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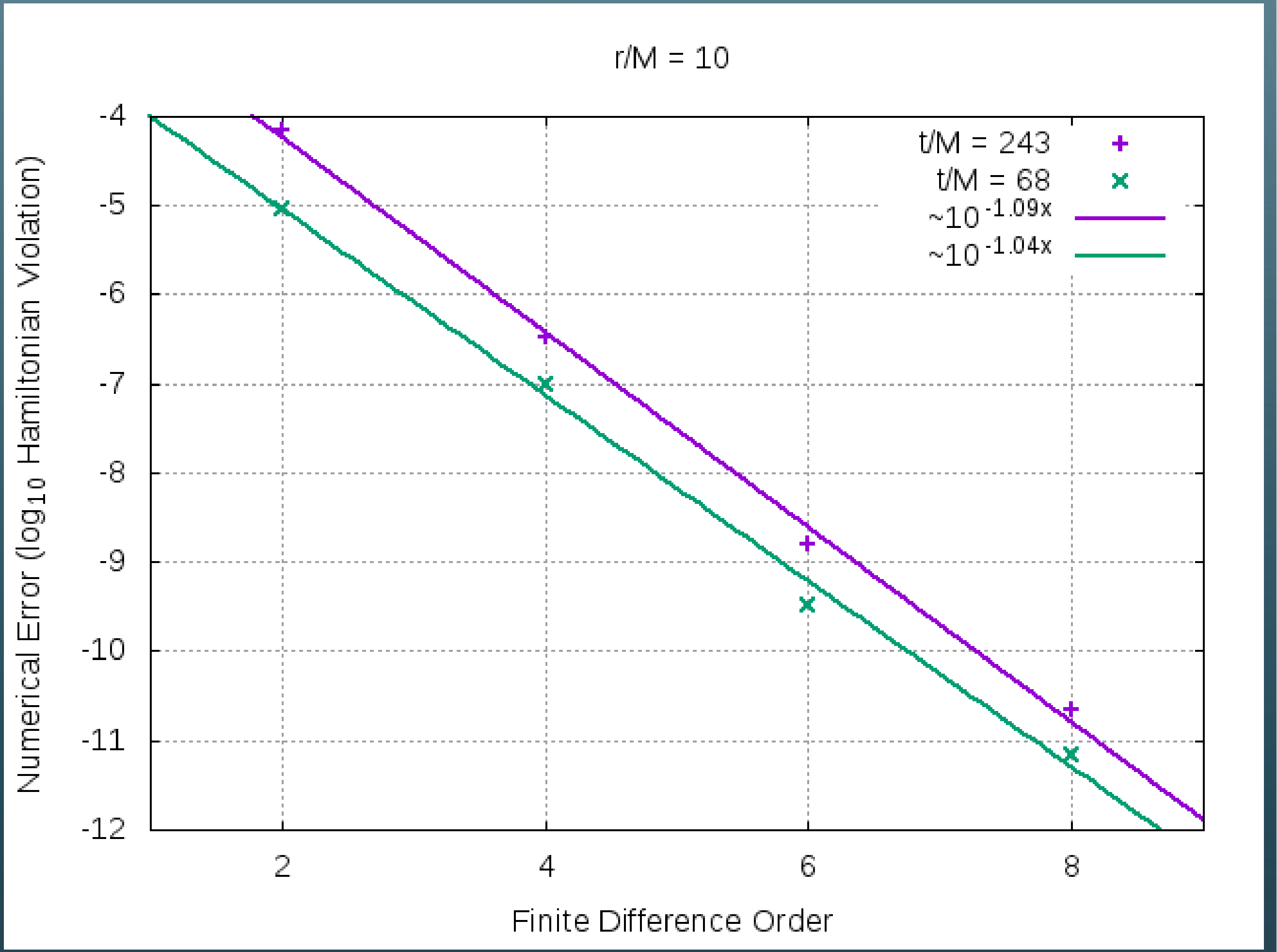
Stable long-term BH evolutions! But does it converge?

SENR Results: Exponential convergence of numerical errors!

Simulating **black hole without excision**:

Numerical errors converge to zero ***exponentially*** with increased polynomial approximation order!



SENR/NRPy: Summary

- Open Source, Open Development → Greater Adoption
  - http://tinyurl.com/senrcode
- Algorithmic Simplicity → More Science Faster
  - Easier to debug & extend
  - Build on tried & true algorithms
    - BSSN in Spherical Polar Coords techniques pioneered by T. Baumgarte et al
      - SENR: Extend ideas to support arbitrary, *dynamical* coords
- Memory Efficiency Is Key Focus: Unlock the Desktop
  - Get public involved → ~10,000x more GW throughput!

- Bottom line: Maximize science with minimal human & computational resources