

# Eigenvalue/Eigenvector Worksheet SOLUTIONS, MATH 521

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Note that these matrices are weird, in that one eigenvalue is always zero. This is done so that when solving the characteristic equation, students do not need to factorize a cubic polynomial. You will also notice an interesting pattern in the eigenvectors and eigenvalues as well.

#1, Matrix #1:

$$\begin{pmatrix} -3 & -1 & 1 \\ 3 & -3 & 3 \\ 3 & 1 & -1 \end{pmatrix}$$

$$-\lambda^3 - 7\lambda^2 - 12\lambda$$

$$\{-4, -3, 0\}$$

$$\begin{pmatrix} -1 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#1, Matrix #2:

$$\begin{pmatrix} -3 & -2 & 2 \\ -1 & 1 & -1 \\ -1 & 6 & -6 \end{pmatrix}$$

$$-\lambda^3 - 8\lambda^2 - 15\lambda$$

$$\{-5, -3, 0\}$$

$$\begin{pmatrix} -1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#2, Matrix #1:

$$\begin{pmatrix} 4 & 4 & -4 \\ -4 & -2 & 2 \\ -4 & 2 & -2 \end{pmatrix}$$

$$16\lambda - \lambda^3$$

$$\{-4, 4, 0\}$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

#2, Matrix #2:

$$\begin{pmatrix} 4 & 9 & -9 \\ 2 & 2 & -2 \\ 2 & 7 & -7 \end{pmatrix}$$

$$-\lambda^3 - \lambda^2 + 20\lambda$$

$$\{-5, 4, 0\}$$

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#3, Matrix #1:

$$\begin{pmatrix} 2 & -5 & 5 \\ -1 & 1 & -1 \\ -1 & 4 & -4 \end{pmatrix}$$

$$-\lambda^3 - \lambda^2 + 6\lambda$$

$$\{-3, 2, 0\}$$

$$\begin{pmatrix} -1 & -2 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#3, Matrix #2:

$$\begin{pmatrix} -2 & 3 & -3 \\ -1 & -1 & 1 \\ -1 & 4 & -4 \end{pmatrix}$$

$$-\lambda^3 - 7\lambda^2 - 10\lambda$$

$$\{-5, -2, 0\}$$

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#4, Matrix #1:

$$\begin{pmatrix} 3 & -7 & 7 \\ -1 & 1 & -1 \\ -1 & 5 & -5 \end{pmatrix}$$

$$-\lambda^3 - \lambda^2 + 12\lambda$$

$$\{-4, 3, 0\}$$

$$\begin{pmatrix} -1 & -3 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#4, Matrix #2:

$$\begin{pmatrix} 4 & 9 & -9 \\ 2 & 2 & -2 \\ 2 & 7 & -7 \end{pmatrix}$$

$$-\lambda^3 - \lambda^2 + 20\lambda$$

$$\{-5, 4, 0\}$$

$$\begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#5, Matrix #1:

$$\begin{pmatrix} 1 & 6 & -6 \\ 1 & 1 & -1 \\ 1 & 6 & -6 \end{pmatrix}$$

$$-\lambda^3 - 4\lambda^2 + 5\lambda$$

$$\{-5, 1, 0\}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#5, Matrix #2:

$$\begin{pmatrix} -4 & 2 & -2 \\ -2 & 2 & -2 \\ -2 & 4 & -4 \end{pmatrix}$$

$$-\lambda^3 - 6\lambda^2 - 8\lambda$$

$$\{-4, -2, 0\}$$

$$\begin{pmatrix} 2 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#6, Matrix #1:

$$\begin{pmatrix} 3 & -6 & 6 \\ 1 & -1 & 1 \\ 1 & 2 & -2 \end{pmatrix}$$

$$9\lambda - \lambda^3$$

$$\{-3, 3, 0\}$$

$$\begin{pmatrix} -1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#6, Matrix #2:

$$\begin{pmatrix} -4 & 2 & -2 \\ 4 & -4 & 4 \\ 4 & -2 & 2 \end{pmatrix}$$

$$-\lambda^3 - 6\lambda^2 - 8\lambda$$

$$\{-4, -2, 0\}$$

$$\begin{pmatrix} -1 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#7, Matrix #1:

$$\begin{pmatrix} 4 & -6 & 6 \\ 4 & -4 & 4 \\ 4 & -2 & 2 \end{pmatrix}$$

$$-\lambda^3 + 2\lambda^2 + 8\lambda$$

$$\{4, -2, 0\}$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#7, Matrix #2:

$$\begin{pmatrix} 3 & 2 & -2 \\ -3 & -1 & 1 \\ -3 & 2 & -2 \end{pmatrix}$$

$$9\lambda - \lambda^3$$

$$\{-3, 3, 0\}$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$

#8, Matrix #1:

$$\begin{pmatrix} -3 & 1 & -1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$$

$$-\lambda^3 - 5\lambda^2 - 6\lambda$$

$$\{-3, -2, 0\}$$

$$\begin{pmatrix} -3 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#8, Matrix #2:

$$\begin{pmatrix} 2 & 5 & -5 \\ -1 & -1 & 1 \\ -1 & 2 & -2 \end{pmatrix}$$

$$-\lambda^3 - \lambda^2 + 6\lambda$$

$$\{-3, 2, 0\}$$

$$\begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#9, Matrix #1:

$$\begin{pmatrix} -3 & -1 & 1 \\ 1 & 1 & -1 \\ 1 & 3 & -3 \end{pmatrix}$$

$$-\lambda^3 - 5\lambda^2 - 6\lambda$$

$$\{-3, -2, 0\}$$

$$\begin{pmatrix} -3 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#9, Matrix #2:

$$\begin{pmatrix} 2 & -5 & 5 \\ -2 & 2 & -2 \\ -2 & 5 & -5 \end{pmatrix}$$

$$-\lambda^3 - \lambda^2 + 6\lambda$$

$$\{-3, 2, 0\}$$

$$\begin{pmatrix} -1 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#10, Matrix #1:

$$\begin{pmatrix} -2 & 1 & -1 \\ -1 & 1 & -1 \\ -1 & 2 & -2 \end{pmatrix}$$

$$-\lambda^3 - 3\lambda^2 - 2\lambda$$

$$\{-2, -1, 0\}$$

$$\begin{pmatrix} 2 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#10, Matrix #2:

$$\begin{pmatrix} 3 & -6 & 6 \\ 3 & -3 & 3 \\ 3 & 0 & 0 \end{pmatrix}$$

$$9\lambda - \lambda^3$$

$$\{-3, 3, 0\}$$

$$\begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#11, Matrix #1:

$$\begin{pmatrix} -3 & 2 & -2 \\ -1 & -1 & 1 \\ -1 & 4 & -4 \end{pmatrix}$$

$$-\lambda^3 - 8\lambda^2 - 15\lambda$$

$$\{-5, -3, 0\}$$

$$\begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#11, Matrix #2:

$$\begin{pmatrix} -5 & 2 & -2 \\ -5 & 5 & -5 \\ -5 & 8 & -8 \end{pmatrix}$$

$$-\lambda^3 - 8\lambda^2 - 15\lambda$$

$$\{-5, -3, 0\}$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#12, Matrix #1:

$$\begin{pmatrix} 2 & -5 & 5 \\ -2 & 2 & -2 \\ -2 & 5 & -5 \end{pmatrix}$$

$$-\lambda^3 - \lambda^2 + 6\lambda$$

$$\{-3, 2, 0\}$$

$$\begin{pmatrix} -1 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#12, Matrix #2:

$$\begin{pmatrix} -4 & 1 & -1 \\ 2 & 2 & -2 \\ 2 & 7 & -7 \end{pmatrix}$$

$$-\lambda^3 - 9\lambda^2 - 20\lambda$$

$$\{-5, -4, 0\}$$

$$\begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#13, Matrix #1:

$$\begin{pmatrix} 3 & 2 & -2 \\ 3 & 1 & -1 \\ 3 & 4 & -4 \end{pmatrix}$$

$$9\lambda - \lambda^3$$

$$\{-3, 3, 0\}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 3 & 1 & 1 \end{pmatrix}$$

#13, Matrix #2:

$$\begin{pmatrix} -4 & 1 & -1 \\ 2 & 2 & -2 \\ 2 & 7 & -7 \end{pmatrix}$$

$$-\lambda^3 - 9\lambda^2 - 20\lambda$$

$$\{-5, -4, 0\}$$

$$\begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#14, Matrix #1:

$$\begin{pmatrix} 2 & 6 & -6 \\ 2 & 2 & -2 \\ 2 & 6 & -6 \end{pmatrix}$$

$$-\lambda^3 - 2\lambda^2 + 8\lambda$$

$$\{-4, 2, 0\}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#14, Matrix #2:

$$\begin{pmatrix} -2 & -3 & 3 \\ -2 & 2 & -2 \\ -2 & 7 & -7 \end{pmatrix}$$

$$-\lambda^3 - 7\lambda^2 - 10\lambda$$

$$\{-5, -2, 0\}$$

$$\begin{pmatrix} -1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#15, Matrix #1:

$$\begin{pmatrix} 5 & -6 & 6 \\ -5 & 5 & -5 \\ -5 & 6 & -6 \end{pmatrix}$$

$$-\lambda^3 + 4\lambda^2 + 5\lambda$$

$$\{5, -1, 0\}$$

$$\begin{pmatrix} -1 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#15, Matrix #2:

$$\begin{pmatrix} -4 & 1 & -1 \\ 2 & -2 & 2 \\ 2 & 1 & -1 \end{pmatrix}$$

$$-\lambda^3 - 7\lambda^2 - 12\lambda$$

$$\{-4, -3, 0\}$$

$$\begin{pmatrix} -2 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#16, Matrix #1:

$$\begin{pmatrix} 4 & 8 & -8 \\ -2 & -2 & 2 \\ -2 & 2 & -2 \end{pmatrix}$$

$$16\lambda - \lambda^3$$

$$\{-4, 4, 0\}$$

$$\begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#16, Matrix #2:

$$\begin{pmatrix} 4 & 4 & -4 \\ -2 & -1 & 1 \\ -2 & 3 & -3 \end{pmatrix}$$

$$16\lambda - \lambda^3$$

$$\{-4, 4, 0\}$$

$$\begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

#17, Matrix #1:

$$\begin{pmatrix} 4 & 9 & -9 \\ -2 & -2 & 2 \\ -2 & 3 & -3 \end{pmatrix}$$

$$-\lambda^3 - \lambda^2 + 20\lambda$$

$$\{-5, 4, 0\}$$

$$\begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#17, Matrix #2:

$$\begin{pmatrix} 4 & -8 & 8 \\ 2 & -2 & 2 \\ 2 & 2 & -2 \end{pmatrix}$$

$$16\lambda - \lambda^3$$

$$\{-4, 4, 0\}$$

$$\begin{pmatrix} -1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#18, Matrix #1:

$$\begin{pmatrix} 4 & 4 & -4 \\ -4 & -2 & 2 \\ -4 & 2 & -2 \end{pmatrix}$$

$$16\lambda - \lambda^3$$

$$\{-4, 4, 0\}$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

#18, Matrix #2:

$$\begin{pmatrix} -3 & 1 & -1 \\ -3 & -3 & 3 \\ -3 & 1 & -1 \end{pmatrix}$$

$$-\lambda^3 - 7\lambda^2 - 12\lambda$$

$$\{-4, -3, 0\}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#19, Matrix #1:

$$\begin{pmatrix} -2 & 3 & -3 \\ 1 & 1 & -1 \\ 1 & 6 & -6 \end{pmatrix}$$

$$-\lambda^3 - 7\lambda^2 - 10\lambda$$

$$\{-5, -2, 0\}$$

$$\begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#19, Matrix #2:

$$\begin{pmatrix} -5 & -2 & 2 \\ -5 & -5 & 5 \\ -5 & -2 & 2 \end{pmatrix}$$

$$-\lambda^3 - 8\lambda^2 - 15\lambda$$

$$\{-5, -3, 0\}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#20, Matrix #1:

$$\begin{pmatrix} 5 & 6 & -6 \\ -5 & -5 & 5 \\ -5 & -4 & 4 \end{pmatrix}$$

$$-\lambda^3 + 4\lambda^2 + 5\lambda$$

$$\{5, -1, 0\}$$

$$\begin{pmatrix} -1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#20, Matrix #2:

$$\begin{pmatrix} 1 & 3 & -3 \\ -1 & -1 & 1 \\ -1 & 1 & -1 \end{pmatrix}$$

$$-\lambda^3 - \lambda^2 + 2\lambda$$

$$\{-2, 1, 0\}$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#21, Matrix #1:

$$\begin{pmatrix} 2 & -4 & 4 \\ -1 & 1 & -1 \\ -1 & 3 & -3 \end{pmatrix}$$

$$4\lambda - \lambda^3$$

$$\{-2, 2, 0\}$$

$$\begin{pmatrix} -1 & -2 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#21, Matrix #2:

$$\begin{pmatrix} 2 & -7 & 7 \\ -1 & 1 & -1 \\ -1 & 6 & -6 \end{pmatrix}$$

$$-\lambda^3 - 3\lambda^2 + 10\lambda$$

$$\{-5, 2, 0\}$$

$$\begin{pmatrix} -1 & -2 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#22, Matrix #1:

$$\begin{pmatrix} 4 & -5 & 5 \\ -4 & 4 & -4 \\ -4 & 5 & -5 \end{pmatrix}$$

$$-\lambda^3 + 3\lambda^2 + 4\lambda$$

$$\{4, -1, 0\}$$

$$\begin{pmatrix} -1 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#22, Matrix #2:

$$\begin{pmatrix} 1 & 6 & -6 \\ -1 & -1 & 1 \\ -1 & 4 & -4 \end{pmatrix}$$

$$-\lambda^3 - 4\lambda^2 + 5\lambda$$

$$\{-5, 1, 0\}$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#23, Matrix #1:

$$\begin{pmatrix} 3 & 8 & -8 \\ 3 & 3 & -3 \\ 3 & 8 & -8 \end{pmatrix}$$

$$-\lambda^3 - 2\lambda^2 + 15\lambda$$

$$\{-5, 3, 0\}$$

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

#23, Matrix #2:

$$\begin{pmatrix} 1 & 4 & -4 \\ -1 & -1 & 1 \\ -1 & 2 & -2 \end{pmatrix}$$

$$-\lambda^3 - 2\lambda^2 + 3\lambda$$

$$\{-3, 1, 0\}$$

$$\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$