## All tangled up in knots

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## Motivation for this talk

- Understanding some foundational concepts
- Differentiate between some knots
- Present a few interesting (and sometimes a little surprising) results
- State a few open problems
- Mention applications


## What is a knot or link?

- Knot - simple closed curve in space

- Link - disjoint union of knots




## Reidemeister moves and ambient isotopy

## Unknot vs Trefoil

- Wolfgang Haken, I96I

Open Problem:
Write a computer program
impletenting Haken's algorithm

- Hass and Lagrias - $\mathbf{2}^{\wedge}(1,000,000,000 n)$

In case you needed more convincing...


Some invariants


## Another example:



## Open problems for p-colorability

- Is there a relationship between $c(K)$ and the largest prime that admits a p-coloring?
- If $K$ is $p$-colorable for what $q$ is $K$ q -colorable ( $\mathrm{q}=\mathrm{kp}$, but what others)?


## Algebraic Knots

- Formed by closure of a tangle


A simple tangle:


## More on tangles...

- Addition

- Multiplication



## Example:



3


32


32-4

## Surprising tangle test


$-232 \stackrel{?}{\cong} 3-23$
$2+\frac{1}{3+\frac{1}{-2}}=2+\frac{2}{5}=\frac{12}{5}=\frac{12}{5}=3-\frac{3}{5}=3+\frac{1}{-2+\frac{1}{3}}$

Nakanishi＇s Conjecture ふかーふが， $+$
Reidemeister Type I，II and III


On a simple tangle


$$
\begin{aligned}
\overline{\overline{-2}} & =\{5 k-2 \mid k \in \mathbb{Z}\} \\
\overline{-1} & =\{5 k-1 \mid k \in \mathbb{Z}\} \\
\overline{0} & =\{5 k \mid k \in \mathbb{Z}\} \\
\overline{1} & =\{5 k+1 \mid k \in \mathbb{Z}\} \\
\overline{2} & =\{5 k+2 \mid k \in \mathbb{Z}\}
\end{aligned}
$$


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## Tangle algebra

- Addition
- Multiplication

$2 \cdot 1$

$2 \cdot 2$


## This shows:

- Any algebraic link is $(2,2)$-equivalent to a link of two or fewer crossings



## Interesting

- $(2,2)$-moves preserve 5-colorability

- (p,q)-moves preserve certain colorabilities too



## Knot composition

- Denoted $K_{1} \# K_{2}$
- Many ways

- Composite vs. Prime


## Big Unsolved Question

- Show that the crossing number of a composite knot is the sum of the crossing numbers of the factor knots, that is,

$$
c\left(K_{1} \# K_{2}\right)=c\left(K_{1}\right)+c\left(K_{2}\right)
$$

1988 (Kauffman, Murasugi and Thistlethwaite) conjecture holds when $\mathrm{K} \# \mathrm{~J}$ is alternating

## Applications

- DNA
- Synthesis of knotted molecules
- Statistical Mechanics
- Graph Theory
- Quantum Computing

Questions?


