All tangled up in knots

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Motivation for this talk

- Understanding some foundational concepts
- Differentiate between some knots
- Present a few interesting (and sometimes a little surprising) results
- State a few open problems
- Mention applications

What is a knot or link?

• Knot - simple closed curve in space





Reidemeister moves and ambient isotopy

Unknot vs Trefoil

- Wolfgang Haken, 1961
 Open Problem:
 Write a computer program
 impletenting Haken's algorithm
- Hass and Lagrias 2^(1,000,000,000n)

In case you needed more convincing...

• Perko Pair



Some invariants

- Crossing number, c(K)
- Unknotting number, u(K)
- Tricolorability







Open problems for p-colorability

- Is there a relationship between c(K) and the largest prime that admits a p-coloring?
- If K is p-colorable for what q is K q-colorable (q=kp, but what others)?















Tangle algebra

- Addition
- Multiplication



 $2 \cdot 1$



 $2 \cdot 2$

This shows:

• Any algebraic link is (2,2)-equivalent to a link of two or fewer crossings







Big Unsolved Question

• Show that the crossing number of a composite knot is the sum of the crossing numbers of the factor knots, that is,

 $c(K_1 \# K_2) = c(K_1) + c(K_2)$

1988 (Kauffman, Murasugi and Thistlethwaite) - conjecture holds when K#J is alternating

Applications

- DNA
- Synthesis of knotted molecules
- Statistical Mechanics
- Graph Theory
- Quantum Computing



