

**Theorem**

Let  $a$  be a nonzero real number, the graph of the equation

$$r \sin \theta = a$$

is a horizontal line  $a$  units above the pole if  $a > 0$  and  $|a|$  units below the pole if  $a < 0$ .

**Theorem**

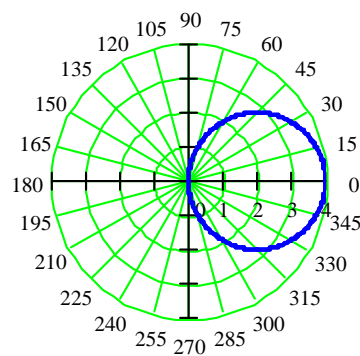
Let  $a$  be a nonzero real number, the graph of the equation

$$r \cos \theta = a$$

is a vertical line  $a$  units to the right of the pole if  $a > 0$  and  $|a|$  units to the left of the pole if  $a < 0$ .

Identify and graph the equation:  $r = 4 \cos \theta$

$$\begin{aligned} r^2 &= 4r \cos \theta \\ x^2 + y^2 &= 4x \\ x^2 - 4x + y^2 &= 0 \\ x^2 - 4x + 4 + y^2 &= 4 \\ (x - 2)^2 + y^2 &= 4 \end{aligned}$$



**Theorem**

Let  $a$  be a positive real number. Then,

$r = 2a \sin \theta$  Circle: radius  $a$ ; center at  $(0, a)$  in rectangular coordinates.

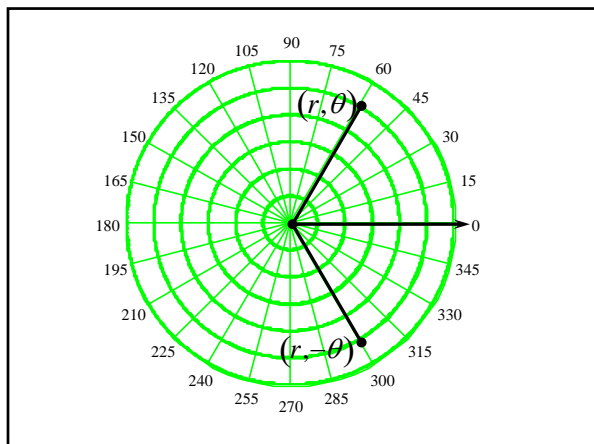
$r = -2a \sin \theta$  Circle: radius  $a$ ; center at  $(0, -a)$  in rectangular coordinates.

**Theorem**

Let  $a$  be a positive real number. Then,

$r = 2a \cos \theta$  Circle: radius  $a$ ; center at  $(a, 0)$  in rectangular coordinates.

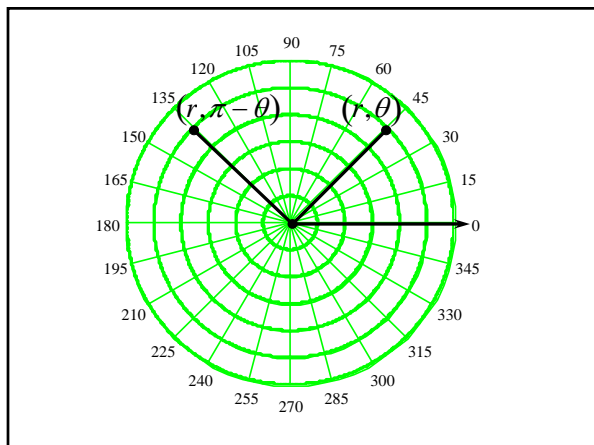
$r = -2a \cos \theta$  Circle: radius  $a$ ; center at  $(-a, 0)$  in rectangular coordinates.



**Theorem Tests for Symmetry**

**Symmetry with Respect to the Polar Axis (x-axis):**

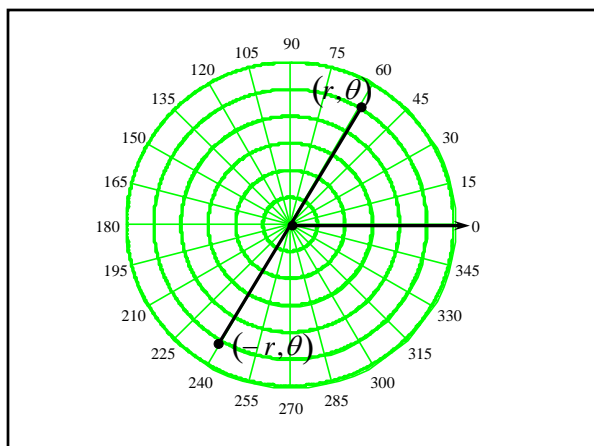
In a polar equation, replace  $\theta$  by  $-\theta$ . If an equivalent equation results, the graph is symmetric with respect to the polar axis.



**Theorem Tests for Symmetry**

**Symmetry with Respect to the Line  $\theta = \pi/2$  (y-axis):**

In a polar equation, replace  $\theta$  by  $\pi - \theta$ . If an equivalent equation results, the graph is symmetric with respect to the line  $\theta = \pi/2$ .



**Theorem Tests for Symmetry**

**Symmetry with Respect to the Pole (Origin):**

In a polar equation, replace  $r$  by  $-r$ . If an equivalent equation results, the graph is symmetric with respect to the pole.