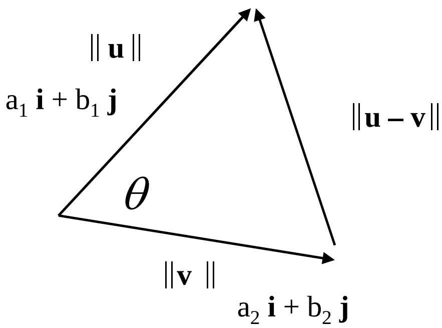


Law of Cosines:

$$\|u-v\|^2 = \|u\|^2 + \|v\|^2 - 2\|u\|\|v\|\cos\theta$$

$$(a_1-a_2)^2 + (b_1-b_2)^2 =$$

$$(a_1^2+b_1^2)+(a_2^2+b_2^2)-2\|u\|\|v\|\cos\theta$$

$$-2a_1a_2 - 2b_1b_2 = -2\|u\|\|v\|\cos\theta$$

$$a_1a_2 + b_1b_2 = \|u\|\|v\|\cos\theta$$

If $\mathbf{v} = a_1\mathbf{i} + b_1\mathbf{j}$ and $\mathbf{w} = a_2\mathbf{i} + b_2\mathbf{j}$ are two vectors, the **dot product** $\mathbf{v}\cdot\mathbf{w}$ is defined as

$$\mathbf{v}\cdot\mathbf{w} = a_1a_2 + b_1b_2$$

If $\mathbf{v} = -2\mathbf{i} + 5\mathbf{j}$ and $\mathbf{w} = 4\mathbf{i} + \mathbf{j}$, find

(a) $\mathbf{v}\cdot\mathbf{w}$ (b) $\mathbf{v}\cdot\mathbf{v}$

(a) $\mathbf{v}\cdot\mathbf{w} = (-2)(4) + (5)(1)$
$= -8 + 5 = -3$

(b) $\mathbf{v}\cdot\mathbf{v} = (-2)(-2) + (5)(5)$
$= 4 + 25 = 29$

Theorem Angle between Vectors

If $\mathbf{u}$ and $\mathbf{v}$ are two nonzero vectors, the angle θ, $0 \le \theta < \pi$, between $\mathbf{u}$ and $\mathbf{v}$ is determined by the formula

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}$$

Find the angle θ between $\mathbf{u} = 2\mathbf{i} - \mathbf{j}$ and $\mathbf{v} = 4\mathbf{i} + 3\mathbf{j}$.

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\,\|\mathbf{v}\|}$$

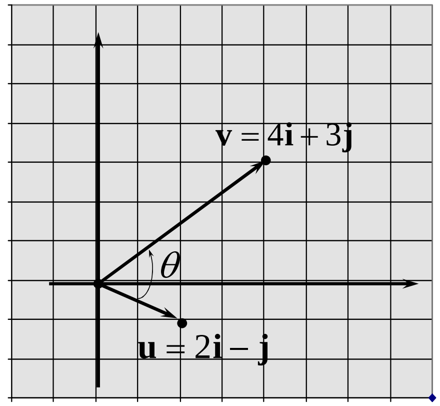
$\mathbf{u}\cdot\mathbf{v} = (2)(4) + (-1)(3) = 8 - 3 = 5$

$\|\mathbf{u}\| = \sqrt{2^2 + (-1)^2} = \sqrt{5}$

$\|\mathbf{v}\| = \sqrt{4^2 + 3^2} = \sqrt{16+9} = \sqrt{25} = 5$

$$\cos\theta = \frac{\mathbf{u}\cdot\mathbf{v}}{\|\mathbf{u}\|\|\mathbf{v}\|} = \frac{5}{\sqrt{5}\cdot 5} = \frac{1}{\sqrt{5}}$$

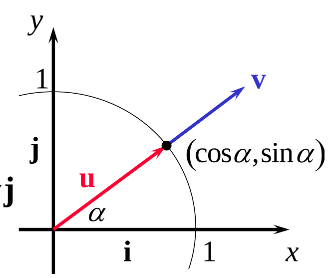
$$\theta \approx 63.4^\circ$$



$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|}$$

$$\mathbf{v} = \|\mathbf{v}\|\mathbf{u}$$

$$\mathbf{u} = \cos\alpha\,\mathbf{i} + \sin\alpha\,\mathbf{j}$$



$$\mathbf{v} = \|\mathbf{v}\|\,(\cos\alpha\,\mathbf{i} + \sin\alpha\,\mathbf{j})$$

Theorem

Two vectors **v** and **w** are orthogonal if and only if

$$\mathbf{v}\cdot\mathbf{w} = 0$$

Determine whether the vectors **v** = 4**i** - **j** and **w** = 2**i** + 8**j** are orthogonal.

$$\mathbf{v}\cdot\mathbf{w} = (4)2 + (-1)8 = 0$$