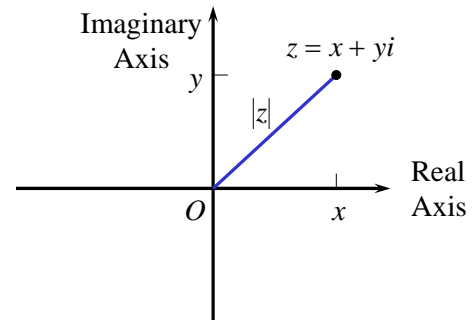


$$x^2 = -1$$

$$i^2 = -1$$

Complex numbers are numbers of the form $a + bi$, where a and b are real numbers. The real number a is called the **real part** of the number $a + bi$; the real number b is called the **imaginary part** of $a + bi$.

Complex Plane



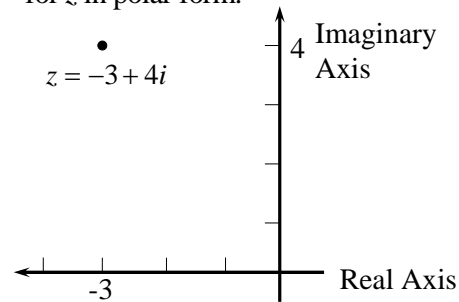
$$z = x + yi = (r \cos \theta) + (r \sin \theta)i$$

Cartesian Form Polar Form

$$z = r(\cos \theta + i \sin \theta)$$

where θ , $0 \leq \theta < 2\pi$, is the **argument of z**

Plot the point corresponding to $z = -3 + 4i$ in the complex plane, and write an expression for z in polar form.



Theorem

Let $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$ and $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$ be two complex numbers. Then

$$z_1 z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)]$$

If $z_2 \neq 0$, then

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)]$$

If $z = 4(\cos 40^\circ + i \sin 40^\circ)$ and $w = 6(\cos 120^\circ + i \sin 120^\circ)$ find
 (a) zw (b) z/w

Theorem DeMoivre's Theorem

If $z = r(\cos\theta + i\sin\theta)$ is a complex number,
then

$$z^n = r^n(\cos n\theta + i\sin n\theta)$$

where $n \geq 1$ is a positive integer.