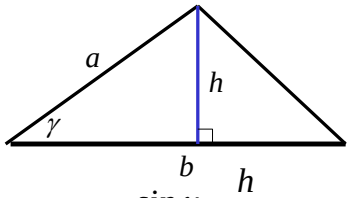
Theorem

The area A of a triangle is

$$A = \frac{1}{2}bh$$

where b is the base and h is the altitude drawn to that base.



$$\sin\gamma = \frac{h}{a}$$

$$h = a\sin\gamma$$

$$A = \frac{1}{2}bh = \frac{1}{2}b(a\sin\gamma) = \frac{1}{2}ab\sin\gamma$$

Theorem

The area A of a triangle equals one-half the product of two of its sides times the sine of its included angle.

$$A = \frac{1}{2}ab\sin\gamma$$

$$A = \frac{1}{2}ac\sin\beta$$

$$A = \frac{1}{2}bc\sin\alpha$$

Find the area A of the triangle for which $a = 5$, $c = 7, \beta = 70^\circ$.

$$A = \frac{1}{2}ac\sin\beta$$

$$= \frac{1}{2}\cdot 5\cdot 7\cdot \sin 70^\circ$$

$$\approx 16.44$$

Theorem Heron's Formula

The area A of a triangle with sides a, b, and c is

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{1}{2}(a+b+c)$.

Find the area of a triangle whose sides are 5, 8, and 11.

Let $a = 5, b = 8, c = 11$

$$s = \frac{1}{2}(a+b+c) = \frac{1}{2}(5+8+11) = 12$$

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{12(12-5)(12-8)(12-11)}$$

$$= \sqrt{336} \qquad = 4\sqrt{21}$$