To solve an oblique triangle means to find the lengths of its sides and the measurements of its angles.

Theorem Law of Sines

For a triangle with sides a, b, c and opposite angles α, β, γ, respectively,

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

We use the Law of Sines to solve CASE 1 (SAA or ASA) and CASE 2 (SSA) of an oblique triangle. The Law of Cosines is used to solve CASES 3 and 4.

CASE 3: Two sides and the included angle are known (SAS).

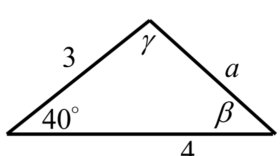
CASE 4: Three sides are known (SSS).

Theorem Law of Cosines

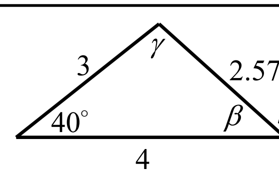
For a triangle with sides a, b, c and opposite angles α, β, γ, respectively.

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$
$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

Solve the triangle: $b = 3, c = 4, \alpha = 40^\circ$ (SAS)



$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$a^2 = 3^2 + 4^2 - 2(3)(4)\cos 40^\circ$$
$$a^2 = 6.614933365$$
$$a \approx 2.57$$



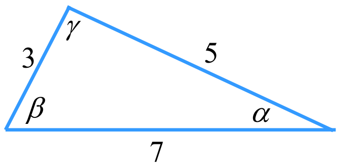
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$\cos\beta = \frac{a^2 + c^2 - b^2}{2ac} = \frac{6.6149 + 4^2 - 3^2}{2(2.57)(4)} \approx 0.6622$$

$$\beta \approx 48.5^\circ$$

$$\gamma = 180^\circ - 40^\circ - 48.5^\circ \qquad \gamma = 91.5^\circ$$

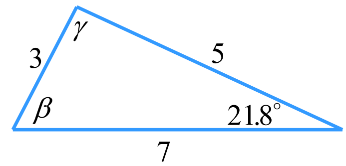
Solve the triangle: $a = 3, b = 5, c = 7$ (SSS)



$$a^2 = b^2 + c^2 - 2bc\cos\alpha \qquad \cos\alpha = \frac{5^2 + 7^2 - 3^2}{2(5)(7)}$$

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc} \qquad \cos\alpha = \frac{65}{70} = \frac{13}{14}$$

$$\alpha \approx 21.8^\circ$$



$$\cos\beta = \frac{a^2 + c^2 - b^2}{2ac} = \frac{3^2 + 7^2 - 5^2}{2(3)(7)} = \frac{33}{42} = \frac{11}{14}$$

$$\beta \approx 38.2^\circ$$

$$\gamma = 180^\circ - 21.8^\circ - 38.2^\circ = 120^\circ$$