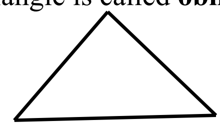
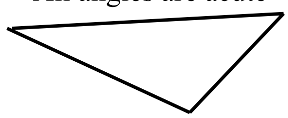
If none of the angles of a triangle is a right angle, the triangle is called **oblique.**



All angles are acute



Two acute angles, one obtuse angle

To solve an oblique triangle means to find the lengths of its sides and the measurements of its angles.

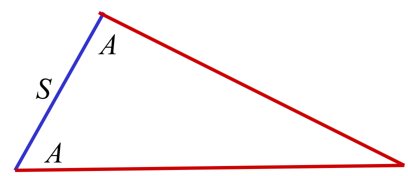
FOUR CASES

CASE 1: One side and two angles are known (SAA or ASA).

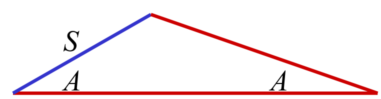
CASE 2: Two sides and the angle opposite one of them are known (SSA).

CASE 3: Two sides and the included angle are known (SAS).

CASE 4: Three sides are known (SSS).

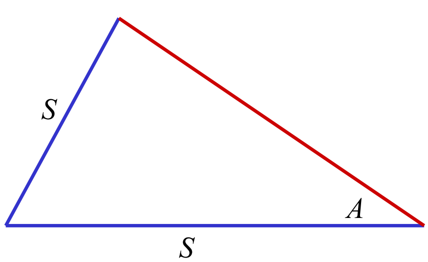


ASA

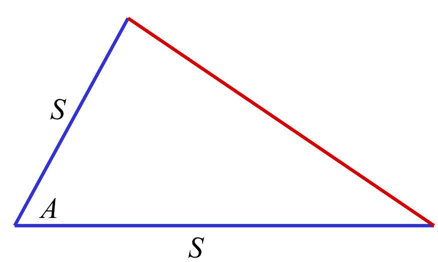


SAA

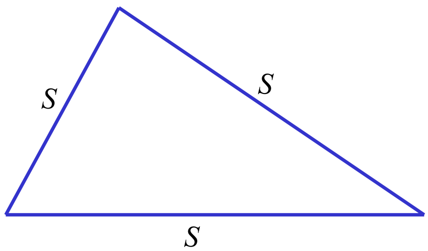
CASE 1: ASA or SAA



CASE 2: SSA



CASE 3: SAS



CASE 4: SSS

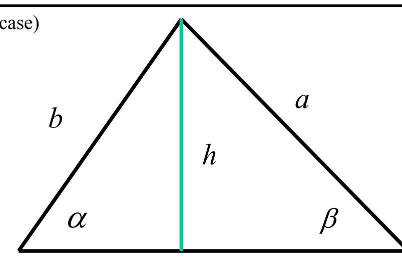
The Law of Sines is used to solve triangles in which Case 1 or 2 holds. That is, the Law of Sines is used to solve SAA, ASA or SSA triangles.

Theorem Law of Sines

For a triangle with sides a, b, c and opposite angles α, β, γ, respectively,

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

Proof (one case)



$\sin\alpha = \frac{opp}{hyp} = \frac{h}{b}$, so $h = b\sin\alpha$

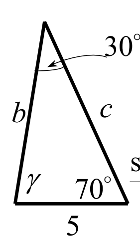
$\sin\beta = \frac{opp}{hyp} = \frac{h}{a}$, so $h = a\sin\beta$

$$h = b\sin\alpha = a\sin\beta$$

$$\frac{\sin\ \alpha}{a} = \frac{\sin\ \beta}{b}$$

$$\alpha + \beta + \gamma = 180^\circ$$

Solve the triangle: $\alpha = 30^\circ, \beta = 70^\circ, a = 5$ (SAA)



$$\alpha + \beta + \gamma = 180^\circ$$

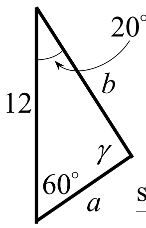
$$30^\circ + 70^\circ + \gamma = 180^\circ$$

$$\gamma = 80^\circ$$

$$\frac{\sin 30^\circ}{5} = \frac{\sin 70^\circ}{b} \qquad \frac{\sin 30^\circ}{5} = \frac{\sin 80^\circ}{c}$$

$$b = \frac{5\sin 70^\circ}{\sin 30^\circ} \approx 9.40 \qquad c = \frac{5\sin 80^\circ}{\sin 30^\circ} \approx 9.85$$

Solve the triangle: $\alpha = 20^\circ, \beta = 60^\circ, c = 12$ (ASA)



$$\alpha + \beta + \gamma = 180^\circ$$

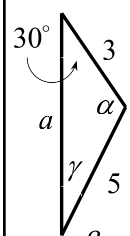
$$20^\circ + 60^\circ + \gamma = 180^\circ$$

$$\gamma = 100^\circ$$

$$\frac{\sin 20^\circ}{a} = \frac{\sin 100^\circ}{12} \qquad \frac{\sin 60^\circ}{b} = \frac{\sin 100^\circ}{12}$$

$$a = \frac{12 \sin 20^\circ}{\sin 100^\circ} \approx 4.17 \qquad b = \frac{12 \sin 60^\circ}{\sin 100^\circ} \approx 10.55$$

Solve the triangle: $b = 5, c = 3, \beta = 30^\circ$ (SSA)



$$\frac{\sin 30^\circ}{5} = \frac{\sin \gamma}{3}$$

$$\sin \gamma = \frac{3 \sin 30^\circ}{5} = 0.3$$

$$\gamma_1 \approx 17.5^\circ \qquad \gamma_2 \approx 162.5^\circ$$

$$\beta + \gamma_2 = 30^\circ + 162.5^\circ = 192.5^\circ > 180^\circ$$

$$\alpha = 180^\circ - 30^\circ - 17.5^\circ \approx 132.5^\circ$$

$$\frac{\sin 132.5^\circ}{a} = \frac{\sin 30^\circ}{5}$$

$$a = \frac{5 \sin 132.5^\circ}{\sin 30^\circ} \approx 7.37$$

$$a \approx 7.37, b = 5, c = 3$$

$$\alpha \approx 132.5^\circ, \beta = 30^\circ, \gamma \approx 17.5^\circ$$

Solve the triangle: $b = 8, c = 10, \beta = 45^\circ$ (SSA)

$$\frac{\sin 45^\circ}{8} = \frac{\sin \gamma}{10}$$

$$\sin \gamma = \frac{10 \sin 45^\circ}{8} \approx 0.88$$

$$\gamma_1 \approx 62.1^\circ \text{ or } \gamma_2 \approx 117.9^\circ$$

$$45^\circ + 62.1^\circ < 180^\circ \qquad 45^\circ + 117.9^\circ < 180^\circ$$

Two triangles!!

Triangle 1: $\gamma_1 \approx 62.1^\circ$

$$\alpha_1 = 180^\circ - 45^\circ - 62.1^\circ \approx 72.9^\circ$$

$$\frac{\sin 72.9^\circ}{a_1} = \frac{\sin 45^\circ}{8}$$

$$a_1 = \frac{8 \sin 72.9}{\sin 45^\circ} \approx 10.81$$

$$a_1 \approx 10.81, b = 8, c = 10$$

$$\alpha_1 \approx 72.9^\circ, \beta = 45^\circ, \gamma_1 \approx 62.1^\circ$$

Triangle 2: $\gamma_2 \approx 117.9^\circ$

$$\alpha_2 = 180^\circ - 45^\circ - 117.9^\circ \approx 17.1^\circ$$

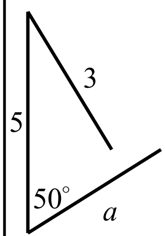
$$\frac{\sin 17.1^\circ}{a_2} = \frac{\sin 45^\circ}{8}$$

$$a_2 = \frac{8 \sin 17.1^\circ}{\sin 45^\circ} \approx 3.33$$

$$a_2 \approx 3.33, b = 8, c = 10$$

$$\alpha_2 \approx 17.1^\circ, \beta = 45^\circ, \gamma_2 \approx 117.9^\circ$$

Solve the triangle: $c = 5, b = 3, \beta = 50^\circ$ (SSA)



$$\frac{\sin 50^\circ}{3} = \frac{\sin \gamma}{5}$$

$$\sin \gamma = \frac{5 \sin 50^\circ}{3}$$

$$\sin \gamma \approx 1.28$$

No triangle with the given measurements!