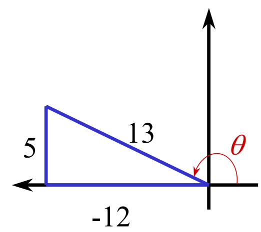
Theorem Double-Angle Formulas

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$\cos 2\theta = 1 - 2\sin^2\theta$$
$$\cos 2\theta = 2\cos^2\theta - 1$$

If $\sin\theta = \frac{5}{13}$, $\pi/2 < \theta < \pi$, find the exact value of

(a) $\sin 2\theta$ (b) $\cos 2\theta$



$$a^2 + b^2 = r^2$$
$$5^2 + b^2 = 13^2$$
$$b^2 = 169 - 25 = 144$$
$$b = -12$$
$$\cos\theta = \frac{b}{r} = \frac{-12}{13}$$

$$\sin\theta = \frac{5}{13} \qquad \cos\theta = -\frac{12}{13}$$

$$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{5}{13}\right)\left(-\frac{12}{13}\right) = -\frac{120}{169}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = \left(-\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$$
$$= \frac{144}{169} - \frac{25}{169} = \frac{119}{169}$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$
$$2\sin^2\theta = 1 - \cos 2\theta$$
$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\cos 2\theta = 2\cos^2\theta - 1$$
$$2\cos^2\theta = 1 + \cos 2\theta$$
$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2} \qquad \cos^2\theta = \frac{1+\cos 2\theta}{2}$$
$$\tan^2\theta = \frac{1-\cos 2\theta}{1+\cos 2\theta}$$

$$\sin^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{2} \qquad \cos^2\frac{\alpha}{2} = \frac{1+\cos\alpha}{2}$$
$$\tan^2\frac{\alpha}{2} = \frac{1-\cos\alpha}{1+\cos\alpha}$$

Theorem Half-Angle Formulas

$$\sin\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{2}}$$

$$\cos\frac{\alpha}{2} = \pm\sqrt{\frac{1+\cos\alpha}{2}}$$

$$\tan\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

where the $+$ or $-$ sign is determined by the quadrant of the angle $\alpha/2$.

If $\csc\alpha = -\frac{3}{2}$, $\pi < \alpha < \frac{3\pi}{2}$, find the exact value of

(a) $\sin\frac{\alpha}{2}$ (b) $\cos\frac{\alpha}{2}$ (c) $\tan\frac{\alpha}{2}$

$\frac{\pi}{2} < \frac{\alpha}{2} < \frac{3\pi}{4}$ so $\frac{\alpha}{2}$ lies in Quadrant II

$\csc\alpha = \frac{r}{b} = \frac{3}{-2}$

$a^2 + b^2 = r^2$, so $a^2 + (-2)^2 = 3^2$

$a^2 = 9 - 4 = 5$ $\qquad a = -\sqrt{5}$

$\cos\alpha = \frac{a}{r} = \frac{-\sqrt{5}}{3}$

(a) $\sin\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{2}} = \sqrt{\frac{1-\left(-\sqrt{5}/3\right)}{2}} = \sqrt{\frac{3+\sqrt{5}}{6}}$

(b) $\cos\frac{\alpha}{2} = -\sqrt{\frac{1+\cos\alpha}{2}} = -\sqrt{\frac{1+\left(-\sqrt{5}/3\right)}{2}} = -\sqrt{\frac{3-\sqrt{5}}{6}}$

(c) $\tan\frac{\alpha}{2} = -\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}} = -\sqrt{\frac{1-\left(-\sqrt{5}/3\right)}{1+\left(-\sqrt{5}/3\right)}} = -\sqrt{\frac{3+\sqrt{5}}{3-\sqrt{5}}}$