



$$(\cos \alpha - \cos \beta)^2 + (\sin \alpha - \sin \beta)^2 =$$

$$(\cos(\alpha - \beta) - 1)^2 + (\sin(\alpha - \beta) - 0)^2$$

$$\cos^2 \alpha - 2 \cos \alpha \cos \beta + \cos^2 \beta +$$

$$\sin^2 \alpha - 2 \sin \alpha \sin \beta + \sin^2 \beta =$$

$$\cos^2(\alpha - \beta) - 2 \cos(\alpha - \beta) + 1 + \sin^2(\alpha - \beta)$$

$$1 + 1 - 2 \cos \alpha \cos \beta - 2 \sin \alpha \sin \beta =$$

$$1 + 1 - 2 \cos(\alpha - \beta)$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

Sum and Difference Formulas for Cosines

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

Find the exact value of $\cos(105^\circ)$.

$$\cos(105^\circ) = \cos(60^\circ + 45^\circ)$$

$$= \cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ$$

$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2} - \sqrt{6}}{4}$$