Two functions f and g are said to be **identically equal** if

$$f(x) = g(x)$$

for every value of x for which both functions are defined. Such an equation is referred to as an **identity**. An equation that is not an identity is called a **conditional equation.**

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Even-Odd Identities

$$\sin(-\theta) = -\sin\theta \qquad \csc(-\theta) = -\csc\theta$$

$$\cos(-\theta) = \cos\theta \qquad \sec(-\theta) = \sec\theta$$

$$\tan(-\theta) = -\tan\theta \qquad \cot(-\theta) = -\cot\theta$$

The directions "*Establish the identity*" means to show, through the use of basic identities and algebraic manipulation, that one side of an equation is the same as the other side of the equation.

Establish the identity:

$$\sin\theta\csc\theta - \cos^2\theta = \sin^2\theta$$

$$\sin\theta\csc\theta - \cos^2\theta = \sin\theta \cdot \frac{1}{\sin\theta} - \cos^2\theta$$

$$= 1 - \cos^2\theta$$

$$= \sin^2\theta$$

Establish the identity:

$$\frac{\cos\theta + 1}{\cos\theta - 1} = \frac{1 + \sec\theta}{1 - \sec\theta}$$

$$\frac{1 + \sec\theta}{1 - \sec\theta} = \frac{1 + \frac{1}{\cos\theta}}{1 - \frac{1}{\cos\theta}} = \left(\frac{1 + \frac{1}{\cos\theta}}{1 - \frac{1}{\cos\theta}}\right)\frac{\cos\theta}{\cos\theta}$$

$$= \frac{\cos\theta + 1}{\cos\theta - 1}$$

Establish the identity:

$$\frac{1+\sin\theta}{1-\sin\theta}-\frac{1-\sin\theta}{1+\sin\theta}=4\tan\theta\sec\theta$$

$$\frac{1+\sin\theta}{1-\sin\theta}-\frac{1-\sin\theta}{1+\sin\theta}$$

$$=\left(\frac{1+\sin\theta}{1-\sin\theta}\right)\left(\frac{1+\sin\theta}{1+\sin\theta}\right)-\left(\frac{1-\sin\theta}{1+\sin\theta}\right)\left(\frac{1-\sin\theta}{1-\sin\theta}\right)$$

$$=\frac{1+2\sin\theta+\sin^2\theta}{1-\sin^2\theta}-\frac{1-2\sin\theta+\sin^2\theta}{1-\sin^2\theta}$$

$$=\frac{1+2\sin\theta+\sin^2\theta}{1-\sin^2\theta}-\frac{1-2\sin\theta+\sin^2\theta}{1-\sin^2\theta}$$

$$=\frac{1+2\sin\theta+\sin^2\theta-\left(1-2\sin\theta+\sin^2\theta\right)}{\cos^2\theta}$$

$$=\frac{1+2\sin\theta+\sin^2\theta-1+2\sin\theta-\sin^2\theta}{\cos^2\theta}$$

$$=\frac{4\sin\theta}{\cos\theta}\cdot\frac{1}{\cos\theta}\quad=4\tan\theta\sec\theta$$

Guidelines for Establishing Identities

- It is almost always preferable to start with the side containing the more complicated expression.
- Rewrite sums or differences of quotients as a single quotient.
- Sometimes rewriting one side in terms of sines and cosines will help.
- Always keep your goal in mind.