A function f is said to be **one-to-one** if, for any choice of numbers x_1 and x_2, $x_1 \neq x_2$, in the domain of f, then $f(x_1) \neq f(x_2)$.

{(1, 1), (2, 4), (3, 9), (4, 16)}

one-to-one

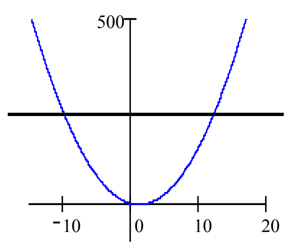
{(-2, 4), (-1, 1), (0, 0), (1, 1)}

not one-to-one

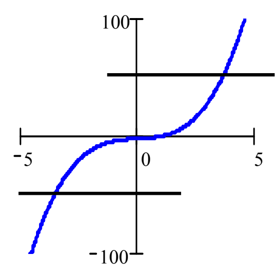
Theorem Horizontal Line Test

If horizontal lines intersect the graph of a function f in at most one point, then f is one-to-one.

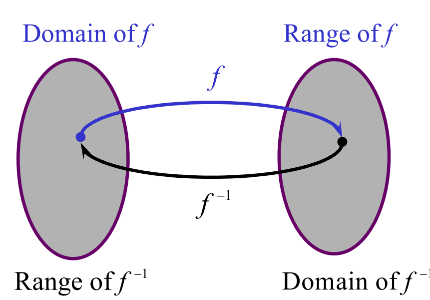
Use the graph to determine whether the function $f(x) = 2x^2 - 5x + 1$ is one-to-one.



Use the graph to determine whether the function $f(x) = x^3 + x - 2$ is one-to-one.



Let f denote a one-to-one function $y = f(x)$. The **inverse of *f***, denoted by f^{-1}, is a function such that $f^{-1}(f(x)) = x$ for every x in the domain of f and $f(f^{-1}(x)) = x$ for every x in the domain of f^{-1}.



Domain of f = Range of f^{-1}

Range of f = Domain of f^{-1}

Theorem

The graph of a function f and the graph of its inverse f^{-1} are symmetric with respect to the line $y = x$.

Find the inverse of $f(x) = \frac{5}{x-3}, x \neq 3$
The function is one-to-one.

$$y = \frac{5}{x-3} \qquad x = \frac{5}{y-3}$$

$$xy - 3x = 5$$

$$xy = 3x + 5$$

$$y = \frac{3x+5}{x} \qquad f^{-1}(x) = \frac{3x+5}{x}$$

Verify $f(x) = \frac{3x+5}{x}$ and $g(x) = \frac{5}{x-3}$ are inverses.

$$f(g(x)) = f\left(\frac{5}{x-3}\right)$$

$$= \frac{3\left(\frac{5}{x-3}\right)+5}{\frac{5}{x-3}} = \frac{3\left(\frac{5}{x-3}\right)+5}{\frac{5}{x-3}} \cdot \frac{x-3}{x-3}$$

$$= \frac{3\left(\frac{5}{x-3}\right)+5}{\frac{5}{x-3}} \cdot \frac{x-3}{x-3} = \frac{15+5(x-3)}{5} = \frac{5x}{5} = x$$

$$g(f(x)) = g\left(\frac{3x+5}{x}\right) = \frac{5}{\frac{3x+5}{x}-3} = \frac{5}{\frac{3x+5}{x}-3} \cdot \frac{x}{x}$$

$$= \frac{5x}{3x+5-3x} = \frac{5x}{5} = x$$