A function f is **increasing** on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) < f(x_2)$.

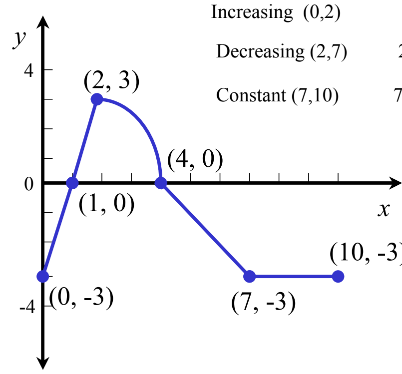
A function f is **decreasing** on an open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) > f(x_2)$.

A function f is **constant** on an open interval I if, for any choice of x in I, the values of $f(x)$ are equal.

Increasing (0,2) $0 < x < 2$

Decreasing (2,7) $2 < x < 7$

Constant (7,10) $7 < x < 10$



A function f has a **local maximum at c** if there is an interval I containing c so that, for all x in I, $f(x) < f(c)$. We call $f(c)$ a **local maximum of f.**

A function f has a **local minimum at c** if there is an interval I containing c so that, for all x in I, $f(x) > f(c)$. We call $f(c)$ a **local minimum of f.**

A function f is **even** if for every number x in its domain the number $-x$ is also in the domain and

$$f(-x) = f(x)$$

A function f is **odd** if for every number x in its domain the number $-x$ is also in the domain and

$$f(-x) = -f(x)$$

Theorem

A function is even if and only if its graph is symmetric with respect to the y-axis. A function is odd if and only if its graph is symmetric with respect to the origin.

Determine whether each of the following functions is even, odd, or neither. Then determine whether the graph is symmetric with respect to the y-axis or with respect to the origin.

(a) $g(z) = -z^2 + 2$

$g(-z) = -(-z)^2+2 = -z^2+2$

$g(z) = g(-z)$

Even function, graph symmetric with respect to the y-axis.

(b) $f(x) = -4x^5 + 3x$

$f(-x) = -4(-x)^5 + 3(-x) = 4x^5 - 3x$

$f(x) \neq f(-x)$

Not an even function

$-f(x) = -(-4x^5 + 3x) = 4x^5 - 3x$

$f(-x) = -f(x)$

Odd function, and the graph is symmetric with respect to the origin.

When functions are defined by more than one equation, they are called **piece-wise defined** functions.

For the following function *f*,

$$f(x) = \begin{cases} x + 3 & -2 \leq x < 1 \\ 3 & x = 1 \\ -x + 3 & x > 1 \end{cases}$$

(a) Find *f*(-1), *f*(1), and *f*(3).
(b) Determine the domain of *f*.
(c) Graph *f*.
(d) Use the graph to find the range of *f*.

(a) *f*(-1) = -1 + 3 = 2

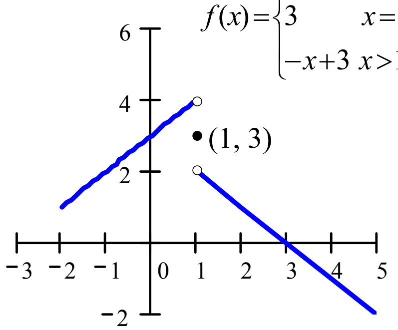
f(1) = 3

f(3) = -3 + 3 = 0

$$f(x) = \begin{cases} x + 3 & -2 \leq x < 1 \\ 3 & x = 1 \\ -x + 3 & x > 1 \end{cases}$$

(b) The domain of *f* is $\{x | x \geq -2\}$ or $[-2, \infty)$.

$$f(x) = \begin{cases} x + 3 & -2 \leq x < 1 \\ 3 & x = 1 \\ -x + 3 & x > 1 \end{cases}$$



(d) From the graph, the range of *f* is $\{y | y < 4\}$ or $(-\infty, 4)$.