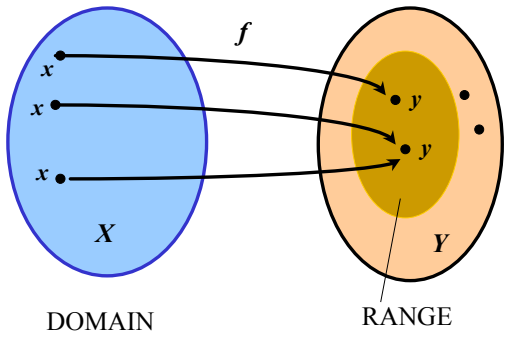
Let X and Y be two nonempty sets of real numbers. A **function** from X into Y is a rule or a correspondence that associates with each element of X a unique element of Y.

The set X is called the domain of the function.

For each element x in X, the corresponding element y in Y is called the **image** of x. The set of all **images** of the elements of the domain is called the **range** of the function.



Which of the following relations are functions?

$\{(1, 1), (2, 4), (3, 9), (-3, 9)\}$

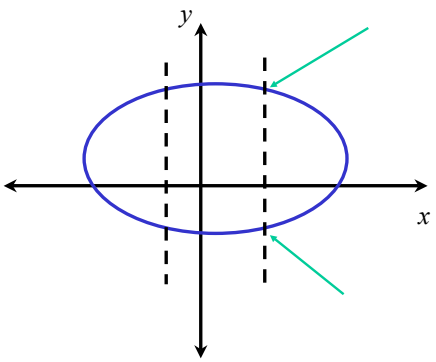
A function

$\{(1, 1), (1, -1), (2, 4), (4, 9)\}$

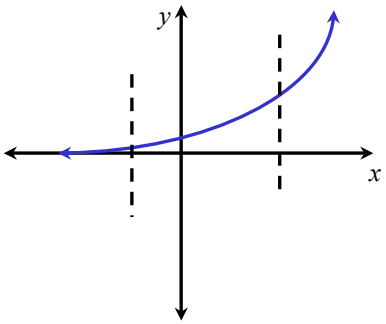
Not a function

Theorem Vertical-Line Test

A set of points in the xy-plane is the graph of a function if and only if any vertical line intersects the graph in at most one point.



Not a function



A function

$$f(x) = 2x^2 + 5$$

$f(x)$ is the number that results when the number x is applied to the rule for f.

$$f(3) = 2(3)^2 + 5 = 23$$

$$f(x+h) = 2(x+h)^2 + 5$$
$$= 2(x^2 + 2xh + h^2) + 5$$
$$= 2x^2 + 4xh + 2h^2 + 5$$

The **domain** of a function f is the set of real numbers such that the rule makes sense.

Find the domain of the following functions:

$g(x) = 3x^3 - 5x + 1$

Domain of g is all real numbers.

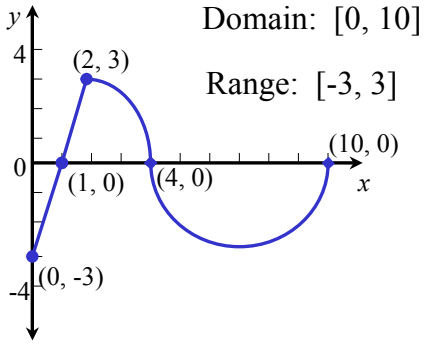
$$s(t) = \frac{4}{t-1}$$

Domain of s is $\{t | t \neq 1\}$.

$$h(z) = \sqrt{z+2}$$

$$z + 2 \geq 0$$
$$z \geq -2$$

Domain of h is $\{z | z \geq -2\}$.



Domain: [0, 10]

Range: [-3, 3]