Section 4.2 The Mean Value Theorem / Rolle's Theorem

Rolle's Theorem: (You must be able to state this)

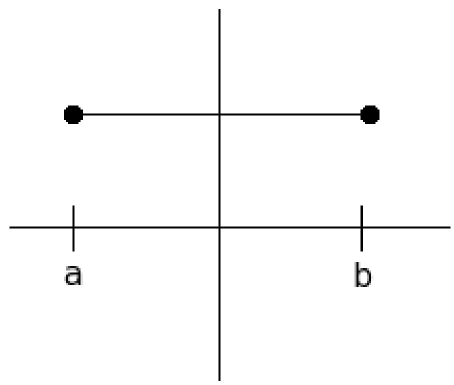
Let *f* be a function that satisfies the following three hypotheses:

1. *f* is continuous on the closer interval $[a , b]$
2. *f* is differentiable on the open interval (a , b)
3. $f(a) = f(b)$

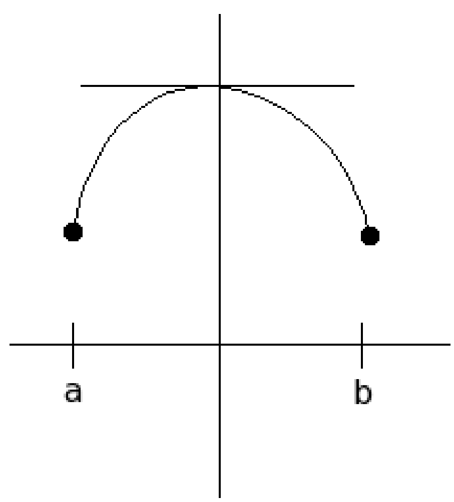
Then there is a number c in (a , b) such that $f'(c) = 0$

Cases:

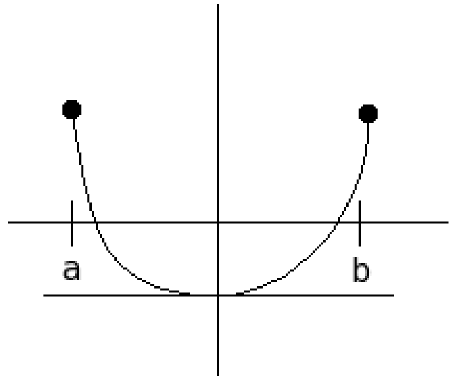
1. Constant Function – *c* is any number between *a* and *b*



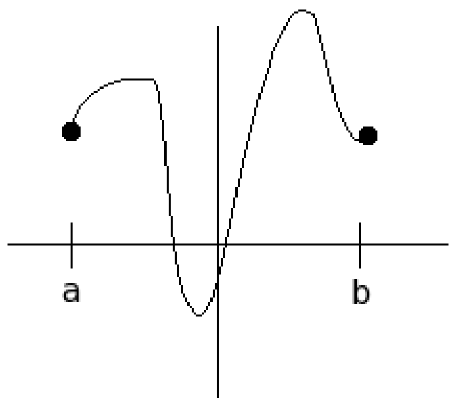
2. A Single Relative Maximum



3. A Single Relative Minimum



4. Multiple Relative Maximums and Minimums



Example 6) Let $f(x) = (x-1)^{-2}$. Show that $f(0) = f(2)$ but there is no number *c* in (0, 2) such that $f'(c)=0$. Why doesn't this contradict Rolle's Theorem?

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	$f(x) = \dfrac{1}{(x-1)^2}$	
0	$f(0) = \dfrac{1}{1} = 1$	So, $f(0) = f(2) = 1$
2	$f(2) = \dfrac{1}{1} = 1$	

$f'(x) = (-2)(x-1)^{-3}(1) = \dfrac{-2}{(x-1)^3}$ So, there is a critical point at $c = 1$ $(f(c) = \text{DNE})$

Check 1 $f(1) = \dfrac{1}{0} = \text{undefined}$ not a max or min?? What does this mean?

Rolle's Theorem fails because there is a vertical asymptote at $x = 1$. This means that *f* is NOT continuous on our interval (0, 2) and that Rolle's Theorem does not apply.

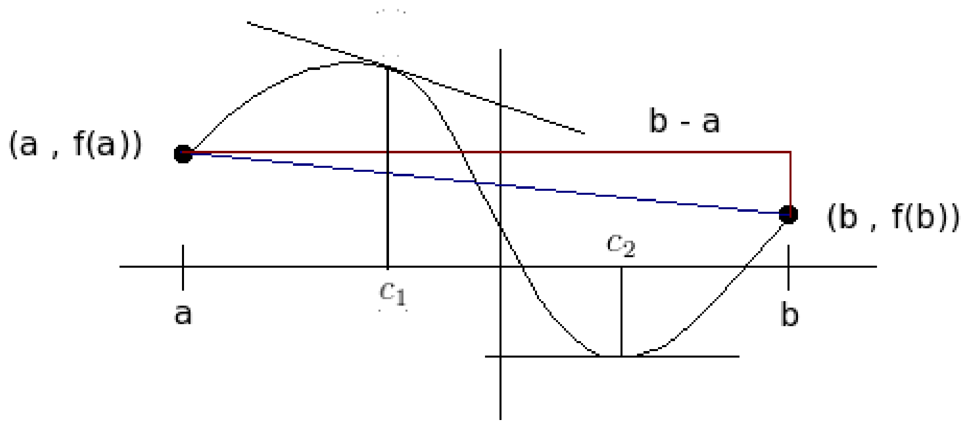
Mean Value Theorem (You must be able to state this)

Let *f* be a function that satisfies the following hypotheses:

1. *f* is continuous on the closed interval [*a* , *b*]
2. *f* is differentiable on the open interval (*a* , *b*)

Then there is a number *c* in (*a* , *b*) such that

$$f'(c) = \frac{f(b)-f(a)}{b-a} \quad \text{or rearranged,} \quad f(b)-f(a)=f'(c)(b-a)$$



Remember: $m = \frac{f(b)-f(a)}{b-a}$

Okay, so what does *f ' (c)* = *m* mean?

Only that I can slide my slope line over my curve and find points along *f* that have the same slope. Those points are *c*. And there may be more than one!

Example Find all the numbers *c* that satisfy the Mean Value Theorem for:

$f(x) = x^3+x-1$ on [0 , 2]

1st is *f* continuous on [0, 2]? Yes, it is a polynomial
2nd is *f* differentiable on (0, 2)? Yes, it is a polynomial

1. $f'(x) = 3x^2+1 \Rightarrow f'(c) = 3c^2+1$
2. Find

pt	$f(x) = x^3+x-1$	
$a = 0$	$0^3+0-1 = -1$	$\Rightarrow f(0) = -1 = f(a)$
$b = 2$	$2^3+2-1 = 9$	$\Rightarrow f(1) = 9 = f(b)$

3. Write out Theorem

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

4. Put in what you know

$$3c^2+1 = \frac{9-(-1)}{2-0} = \frac{10}{2} = 5$$

5. Simplify

$$3c^2+1 = 5$$

$$3c^2 = 4$$

$$c^2 = \frac{4}{3}$$

$$c = \pm\frac{2}{\sqrt{3}}$$

Only $\frac{+2}{\sqrt{3}}$ is in the interval [0 , 2]

So, $\boxed{c = \frac{2}{\sqrt{3}}}$