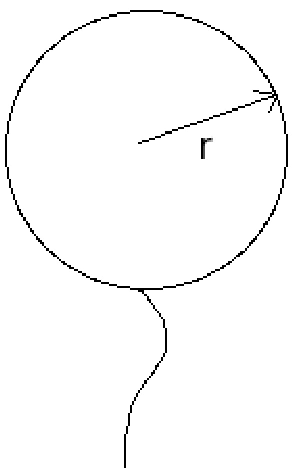
Section 2.7 Related Rates (Word Problems)

The idea is to compute the rate of change of one quantity in terms of the rate of change of another quantity.

(pg. 127) **Example**: Air is being pumped into a spherical balloon so that its volume increases at a rate of 100 cm^3 / s. How fast is the radius of the balloon increasing when the diameter is 50 cm?

Step 1 & 2: Draw a picture and list what you know.



$\frac{dV}{dt} = 100 \frac{cm^3}{s}$ (units tell variables)

$d = 2\text{r} = 50\,cm$ $r = 25\,cm$

Find: $\frac{dr}{dt}$

Step 3: Write an equation relating the variables.

$$V = \frac{4}{3}\pi r^3$$

Step 4: Take derivatives of both sides (implicitly).

$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)$$
$$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi(\cancel{3}r^2)\frac{dr}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Step 5 Solve for what you are trying to find.

$$\frac{dr}{dt} = \frac{1}{4\pi r^2}\cdot\frac{dV}{dt}$$

Step 6 Substitute in what you know.

$$\frac{dr}{dt} = \frac{1}{4\pi(25)^2}\cdot 100 = \frac{25}{\pi(25)^2} = \frac{1}{25\pi}$$

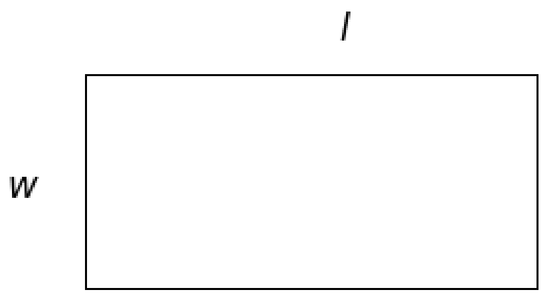
Step 7: Check the units.

$$\frac{dr}{dt} \approx \frac{cm}{s} \qquad \frac{1}{25\pi}\frac{cm}{s}$$

Note: There are 5 very nice examples worked out in the textbook.

(pg. 131, #4) **Example**: The length of a rectangle is increasing at a rate of 8 cm / s and its width is increasing at a rate of 3 cm / s. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing?

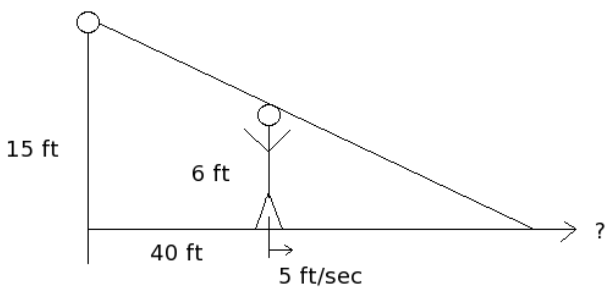
1. Draw Picture:

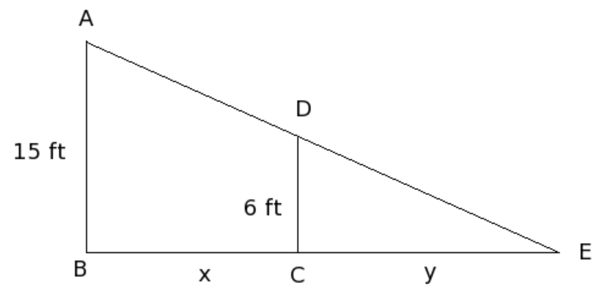


2. List what you know:
$$\frac{dl}{dt} = 8\frac{cm}{s} \qquad \frac{dw}{dt} = 3\frac{cm}{s}$$
3. Write an equation:
$$A = l \cdot w$$
4. Take derivatives:
$$\frac{d}{dt}(A) = \frac{d}{dt}(lw)$$
$$\frac{dA}{dt} = l\frac{dw}{dt} + w\frac{dl}{dt}$$
5. Solve for what you need:
Already in the form we need.
6. Substitute in:
$$\frac{dA}{dt} = l\frac{dw}{dt} + w\frac{dl}{dt}$$
$$\frac{dA}{dt} = 20\,cm \cdot 3\frac{cm}{s} + 10\,cm \cdot 8\frac{cm}{s}$$
$$\frac{dA}{dt} = 60\frac{cm^2}{s} + 80\frac{cm^2}{s}$$
$$\frac{dA}{dt} = 140\frac{cm^2}{s}$$
7. Check units:
$$\frac{dA}{dt} \approx \frac{cm^2}{s}$$ all good.

Example: A street light is mounted at the top of a 15 ft tall pole. A man 6 ft tall walks away from the pole with a speed of 5 ft / sec along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?

1. Draw Picture:





2. List what you know:
   $\frac{dx}{dt} = 5\frac{ft}{s}$ when $x = 40$ ft
3. Write an equation:
   Notice Similar Triangles ABE and DCE.
   So: $\frac{15}{6} = \frac{x+y}{y}$

   $15y = 6(x+y)$
   $15y = 6x + 6y$
   $9y = 6x$
   $y = \frac{2}{3}x$

   Aside: $y = \frac{2}{3}(40) = \frac{80}{3}$
4. Take derivatives:
   (tough part)
   Key: The tip of the shadow moves at a rate of $\frac{d}{dt}(x+y)$
   $\frac{d}{dt}(x+\frac{2}{3}x) = \frac{d}{dt}(\frac{5}{3}x) = \frac{5}{3}\frac{dx}{dt}$
5. Solve for what you need:
   Already in the form we need
6. Substitute in:
   $\frac{d}{dt}(x+y) = \frac{5}{3}\left(5\frac{ft}{s}\right) = \frac{25}{3}\frac{ft}{s}$
7. Check units:
   $\frac{d}{dt}(x+y) = \frac{ft}{s}$ OK.