Section 2.2 The Derivative as a Function

Last time we looked at the derivative of a function at a particular value *a* and we evaluated:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

Now, let's just think of *a* as being variable (i.e. any number) say *x*.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Now, *f ' (x)* is the derivative of *f* at any *x*. But the math works the same way.

Example: Given $f(x) = \sqrt{x+1}$ find *f ' (x)* using the limit definition.

$$f(x+h) = \sqrt{x+h+1}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h}$$

rationalize: $\lim_{h \to 0} \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \cdot \frac{\sqrt{x+h+1} + \sqrt{x+1}}{\sqrt{x+h+1} + \sqrt{x+1}}$

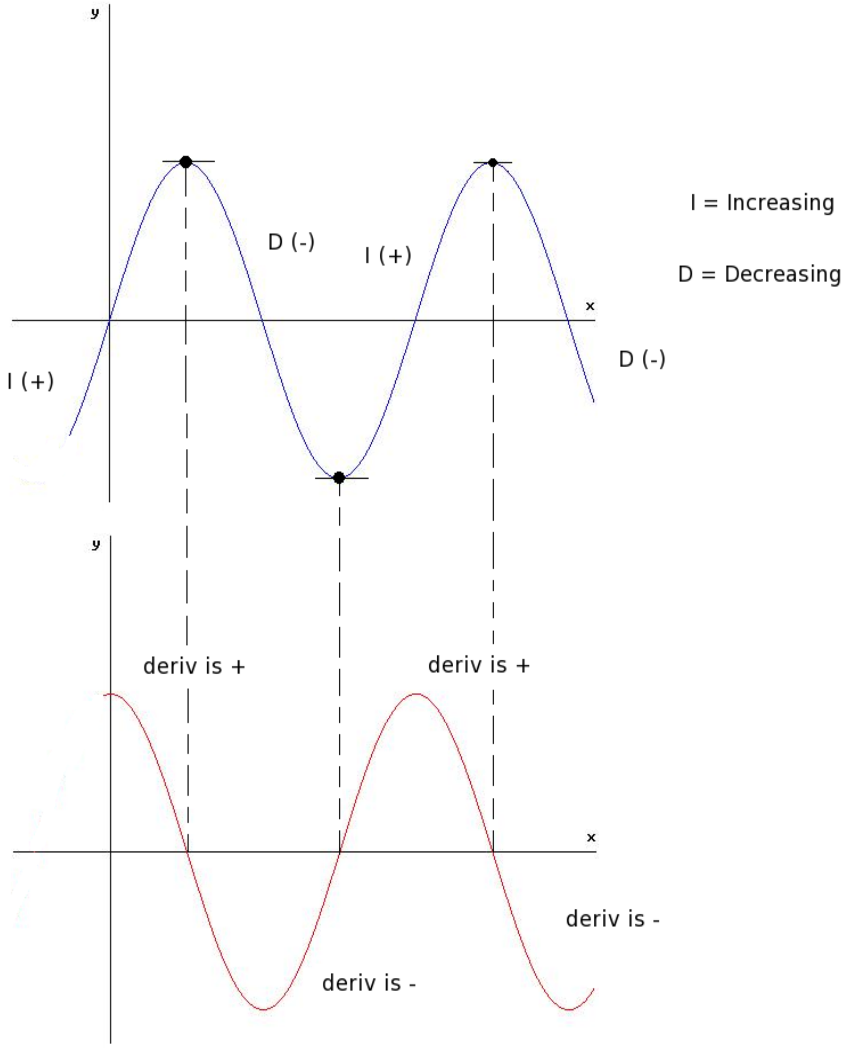
simplify: $\lim_{h \to 0} \frac{(x+h+1) - (x+1)}{h(\sqrt{x+h+1} + \sqrt{x+1})}$

cancel: $\lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h+1} + \sqrt{x+1})}$

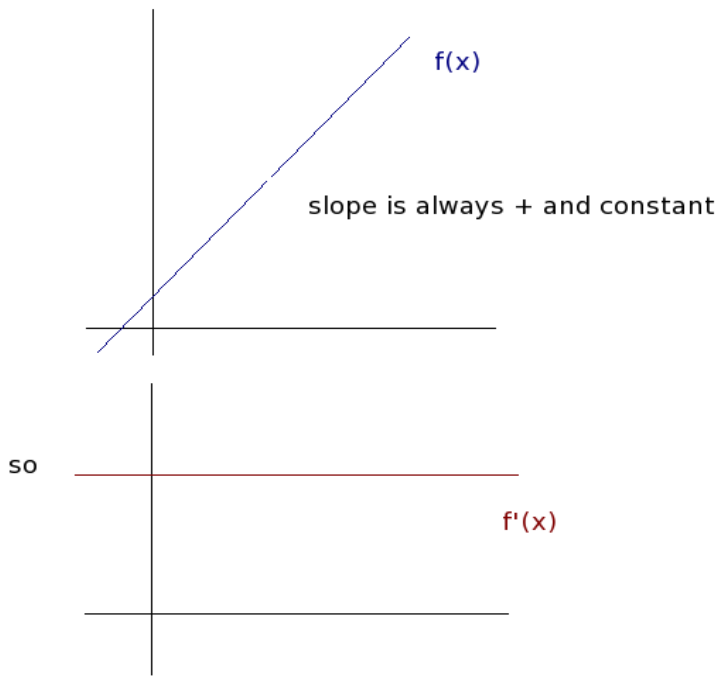
substitute in *h=0*: $\frac{1}{\sqrt{x+1} + \sqrt{x+1}} = \frac{1}{2\sqrt{x+1}} = f'(x)$

Another important thing we need to be able to do is estimate derivatives graphically (without knowing any equations, formulae, or functions).

Example: How might the derivative look? Use the nature of slope and recall derivatives represent instantaneous slopes. What slopes can we draw from f(x)?


y
x
I (+)
D (-)
I (+)
D (-)
I = Increasing
D = Decreasing
y
x
deriv is +
deriv is +
deriv is -
deriv is -

What about?

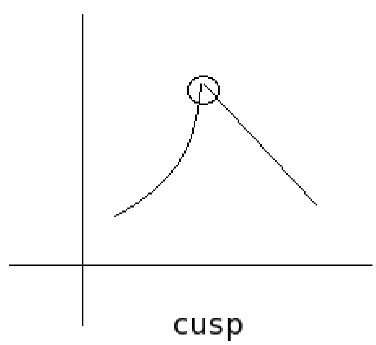


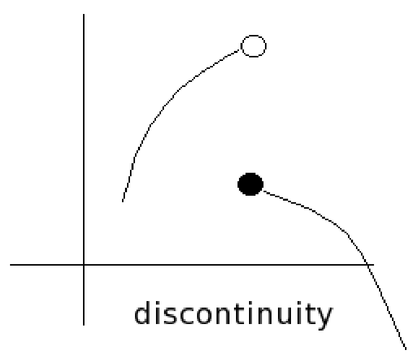
A word about notation.   All of these things mean the same thing:

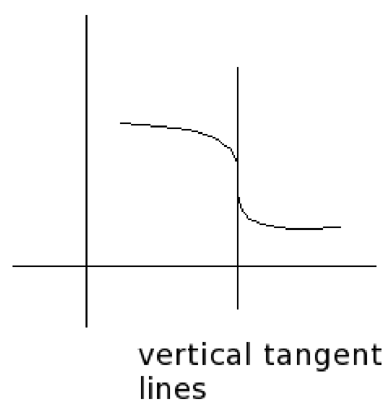
$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx}\left(f(x)\right)$$

Definition: A function *f* is differentiable at *a* if *f ' (a)* exists.
A function *f* is differentiable on an open interval *(a, b)* if it is differentiable at every number in the interval.

When will a function not be differentiable?







An abrupt change in direction

$L \neq R$

$L \neq R$

$$\lim_{x \to a} |f(x)| = \infty$$

Theorem: If *f* is differentiable at *a*, then *f* is continuous at *a*.
(Note, however, that continuity does not guarantee differentiability)

Derivatives are fun because they describe new functions. That means derivatives can have derivatives and their derivatives can have derivatives and so on.

$f(x)$	
$f'(x)$	1st derivative
$f''(x)$	2nd derivative
$f'''(x)$	3rd derivative
$f^{(4)}(x)$	4th derivative

Because we know how to associate derivatives with units, each level of differentiation can be related back to a physical construct or meaning.

Remember:	$s(t)$	[m]
	$v(t) = s'(t)$	[m/s]
	$a(t) = v'(t) = s''(t)$	[m/s^2]