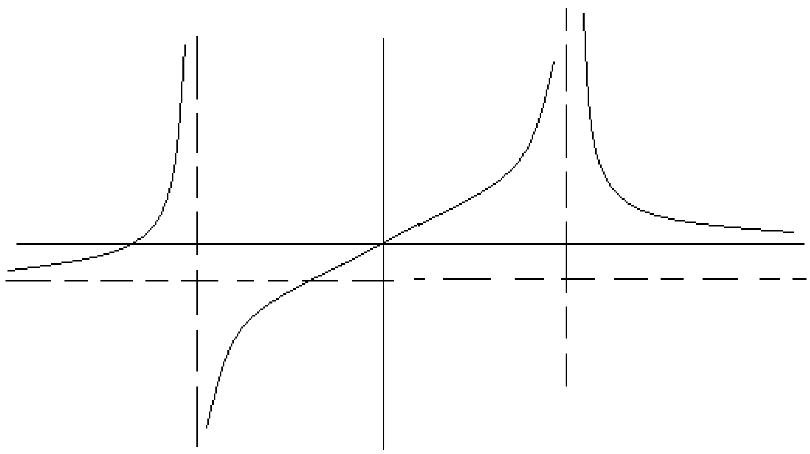
Section 1.6 Limits Involving Infinity

We will be looking at asymptotes:

$\lim_{x \to a} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = L$

Special Vertical Horizontal

When does this happen?



Notation:

Vertical:

$\lim_{x \to a} f(x) = \text{DNE}$

$\lim_{x \to b} f(x) = +\infty$

$\lim_{x \to a^+} f(x) = -\infty$

$\lim_{x \to a^-} f(x) = +\infty$

$\lim_{x \to b^+} f(x) = +\infty$

$\lim_{x \to b^-} f(x) = +\infty$

Horizontal:

$\lim_{x \to \infty} f(x) = 0$

$\lim_{x \to \infty} f(x) = c$

How do you choose $+\infty$ or $-\infty$ without a picture?

Example: $\lim_{x \to 5^-} \frac{6}{x-5} = -\infty$ $\lim_{x \to 5^+} \frac{6}{x-5} = +\infty$

Note: $x-5 = 0$
$x = 5$

Check 5^-, choose say 4.99 and look at: $\frac{6}{x-5} = \frac{+}{-} = -$ tells me $-\infty$.

Check 5^+, choose say 5.01 and look at: $\frac{6}{x-5} = \frac{+}{+} = +$ tells me $+\infty$.

Example: $\lim_{x\to 3^+} \frac{t^2-4t-5}{t^2-t-6} = \lim_{x\to 3} \frac{(t-5)(t+1)}{(t-3)(t+2)} = -\infty$

How?

Check 3^+, choose say 3.01: $\frac{(t-5)(t+1)}{(t-3)(t+2)} = \frac{(-)(+)}{(+)(+)} = (-)$ tells me $-\infty$.

Example: $\lim_{x\to 2^-} \frac{(t-5)(t+1)}{(t-3)(t+2)} = +\infty$

Check 2^-, choose say 1.99: $\frac{(t-5)(t+1)}{(t-3)(t+2)} = \frac{(-)(+)}{(-)(+)} = (+)$ tells me $+\infty$.

Horizontal Asymptotes

Know: $\lim_{x\to\infty} \frac{1}{x^n} = 0$ and $\lim_{x\to -\infty} \frac{1}{x^n} = 0$ and everything else falls out.

Example: $\lim_{x\to\infty} \frac{t^2+2}{t^3+t^2-1} = ?$

$$\lim_{x\to\infty} \frac{t^2+2}{t^3+t^2-1} \cdot \frac{\frac{1}{t^3}}{\frac{1}{t^3}} = \lim_{x\to\infty} \frac{\frac{t^2}{t^3}+\frac{2}{t^3}}{\frac{t^3}{t^3}+\frac{t^2}{t^3}-\frac{1}{t^3}} = \frac{\lim_{x\to\infty} \frac{t^2}{t^3} + \lim_{x\to\infty} \frac{2}{t^3}}{\lim_{x\to\infty} \frac{t^3}{t^3} + \lim_{x\to\infty} \frac{t^2}{t^3} - \lim_{x\to\infty} \frac{1}{t^3}}$$

$$= \frac{\lim_{x\to\infty} \frac{1}{t} + \lim_{x\to\infty} \frac{2}{t^3}}{\lim_{x\to\infty} \frac{1}{1} + \lim_{x\to\infty} \frac{1}{t} - \lim_{x\to\infty} \frac{1}{t^3}} = \frac{0+2(0)}{1+0-0} = \frac{0}{1} = \mathbf{0}$$

Example: $\lim_{x\to\infty} \sqrt{x^2+1} - x = ?$

$$\lim_{x\to\infty} \left(\sqrt{x^2+1} - x\right) \cdot \frac{\sqrt{x^2+1} + x}{\sqrt{x^2+1} + x} = \lim_{x\to\infty} \frac{x^2+1-x^2}{\sqrt{x^2+1} + x} = \lim_{x\to\infty} \frac{1}{\sqrt{x^2+1} + x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$$

$$= \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{\sqrt{x^2+1}}{x} + 1} = \lim_{x\to\infty} \frac{\frac{1}{x}}{\sqrt{\frac{x^2+1}{x^2}} + 1} = \lim_{x\to\infty} \frac{\frac{1}{x}}{\sqrt{1+\frac{1}{x^2}} + 1} = \frac{0}{\sqrt{1+0}+1}$$

$$= \mathbf{0}$$

Final Comments:

What does $\lim_{x\to\infty} f(x) = \infty$ mean?
As *x* gets really big, so does *f(x)*.

Example: $\lim_{x\to\infty} \frac{x^3-2x+3}{5-2x^2} = ?$

$$\lim_{x\to\infty} \frac{x^3-2x+3}{5-2x^2} = \lim_{x\to\infty} \frac{x^3-2x+3}{5-2x^2} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}} = \lim_{x\to\infty} \frac{x-\frac{2}{x}+\frac{3}{x^2}}{\frac{5}{x^2}-2}$$

$$= \frac{\lim_{x\to\infty} x - \lim_{x\to\infty} \frac{2}{x} + \lim_{x\to\infty} \frac{3}{x^2}}{\lim_{x\to\infty} \frac{5}{x^2} - \lim_{x\to\infty} 2} = \frac{\infty-0+0}{0-2} = \boldsymbol{\infty}$$

Example: $\lim_{x\to-\infty} \left(x^2-x^4\right) = ?$

$$\lim_{x\to-\infty} \left(x^2-x^4\right) = \lim_{x\to-\infty} x^2\left(1-x^2\right) = \lim_{x\to-\infty} \left(x^2\right) \cdot \lim_{x\to-\infty} \left(1-x^2\right) = (\infty)(1-\infty)$$

$$= (\infty)(-\infty) = \boldsymbol{-\infty}$$

*Note: it's OK to multiply by infinity but not to add or subtract it.