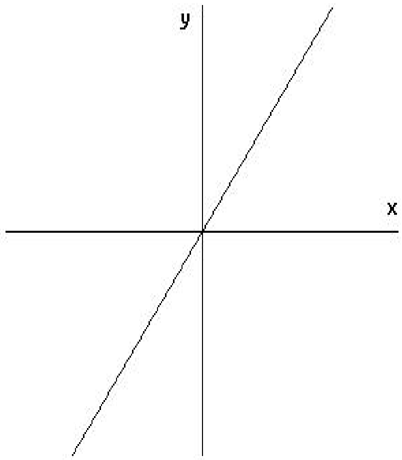
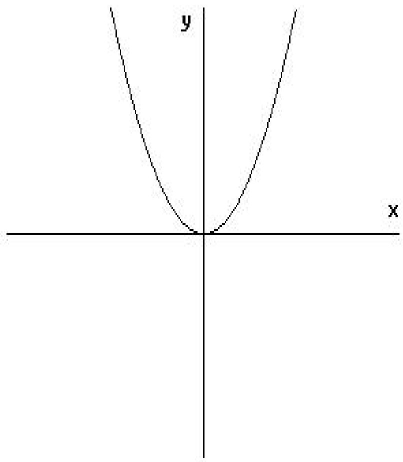
Section 1.2 A Catalog of Essential Functions

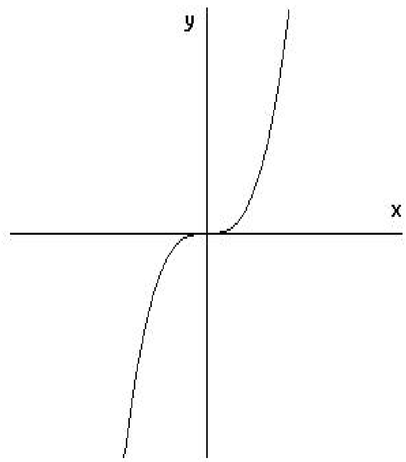
Power Functions $f(x) = x^n$ where n is an integer
some examples include:



$f(x) = x$
D: $(-\infty, \infty)$, R: $(-\infty, \infty)$

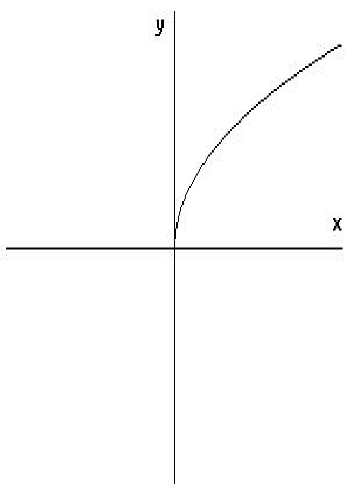


$f(x) = x^2$
D: $(-\infty, \infty)$, R: $[0, \infty)$

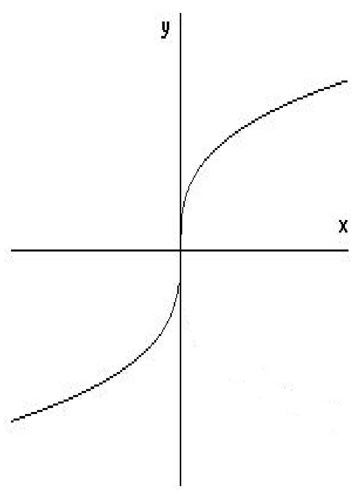


$f(x) = x^3$
D: $(-\infty, \infty)$, R: $(-\infty, \infty)$

Root Functions $f(x) = x^{\frac{1}{n}}$ or $f(x) = \sqrt[n]{x}$
some examples include

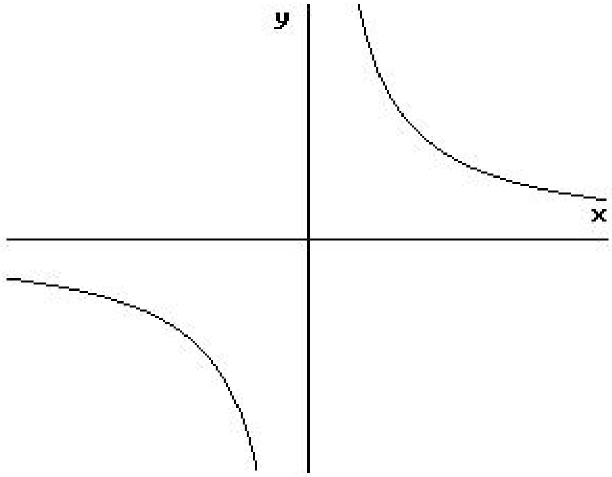


$f(x) = \sqrt{x}$
D: $[0, \infty)$, R: $[0, \infty)$



$f(x) = \sqrt[3]{x}$ or $x^{1/3}$
D: $(-\infty, \infty)$, R: $(-\infty, \infty)$

Reciprocal Functions

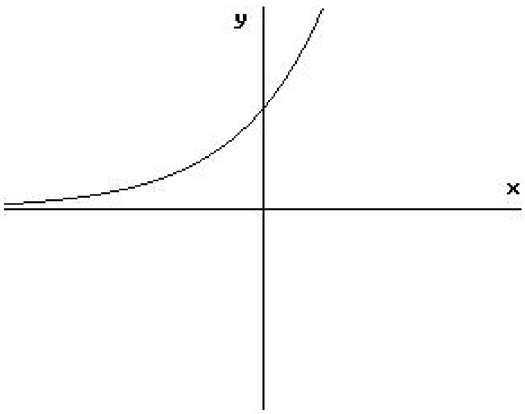


$$f(x)=\frac{1}{x}$$

D: $(-\infty,0)\cup(0,\infty)$, R: $(-\infty,0)\cup(0,\infty)$

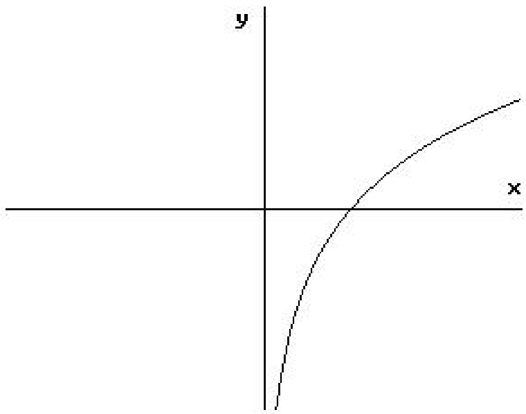
Exponential Functions and Logarithmic Functions

some examples include:



$f(x)=e^x$

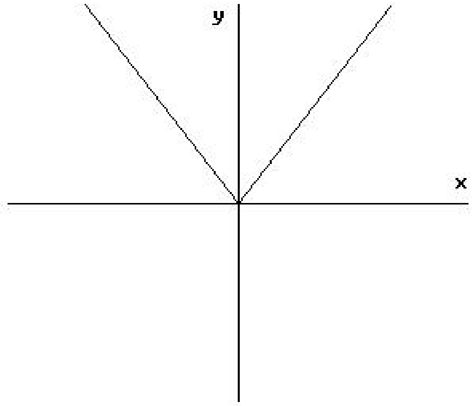
D: $(-\infty,\infty)$, R: $(0,\infty)$



$f(x)=\ln(x)$

D: $(0,\infty)$, R: $(-\infty,\infty)$

Absolute Value Functions

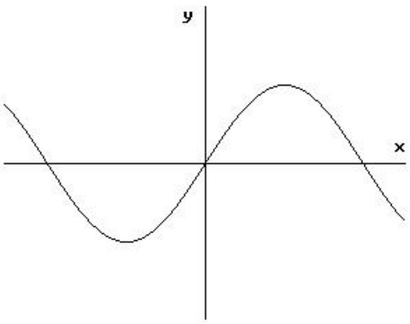


$f(x)=|x|$

D: $(-\infty,\infty)$, R: $[0,\infty)$

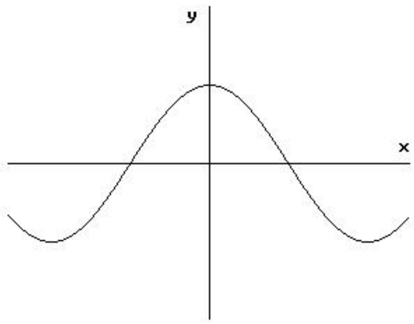
Trigonometric Functions

some examples include:



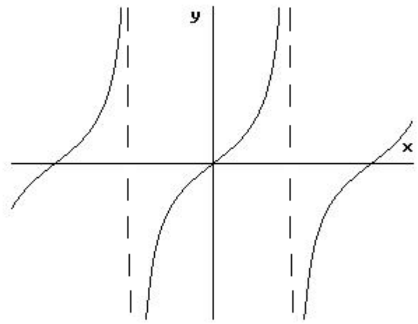
$f(x)=\sin(x)$

D: $(-\infty,\infty)$, R: $[-1,1]$



$f(x)=\cos(x)$

D: $(-\infty,\infty)$, R: $[-1,1]$



$f(x)=\tan(x)$

D: $x\in\mathbb{R}, x\neq\frac{2n+1}{2}\pi$,

n = integer

R: $(-\infty,\infty)$

Just knowing these basic shapes, domains, and ranges, we can build many, many more functions and know what they look like as well—all without a calculator.

f(x)	$a\,f\left(b(x+c)\right)+d$
Basic Function	*a* is the vertical scale factor *b* is the horizontal scale factor *c* is the horizontal shift *d* is the vertical shift

Note on the addition of negative signs

on the vertical scale factor, -a, flips over x-axis

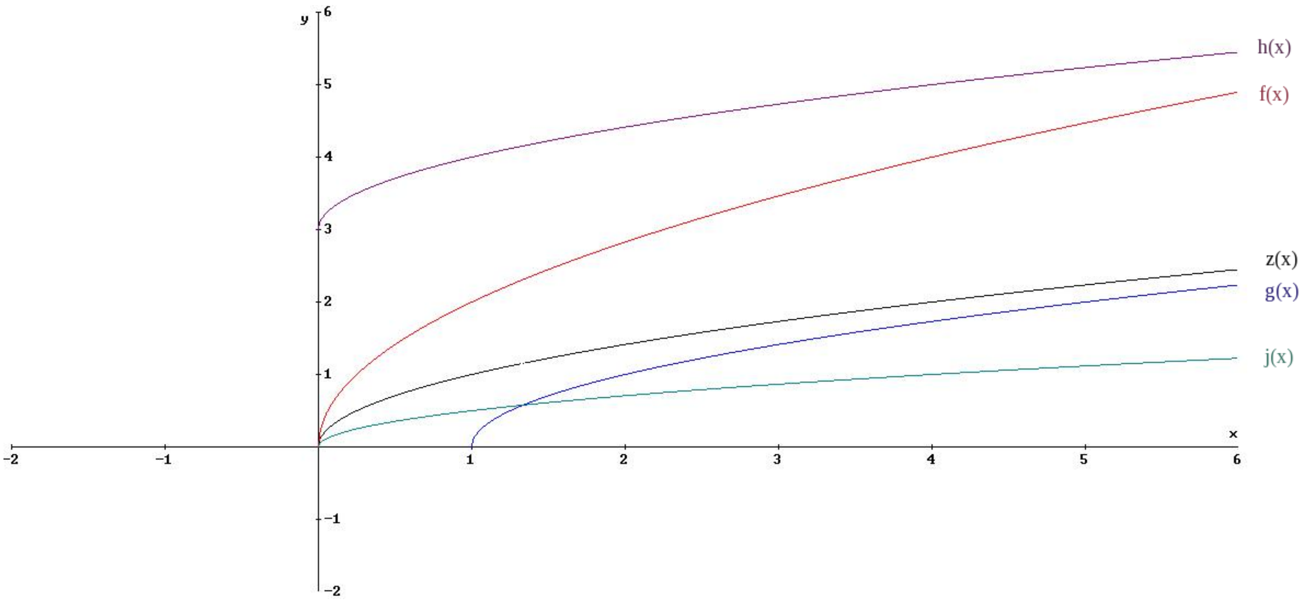
on the horizontal scale factor, -b, flips over y-axis

Base Function $z(x)=\sqrt{x}$

Example Graph:

$$f(x) = 2\sqrt{x} = 2z(x)$$
$$g(x) = \sqrt{x-1} = z(x-1)$$
$$h(x) = \sqrt{x}+3 = z(x)+3$$
$$j(x) = \sqrt{\frac{1}{4}x} = z(\frac{1}{4}x)$$

Vertical Shift = 2
Horizontal Shift (- right, + left)
Vertical Shift (- down, + up)
Horizontal Scale = ¼



1st Sketch a base graph
2nd Identify alteration, sketch
If more than one alteration, do in steps

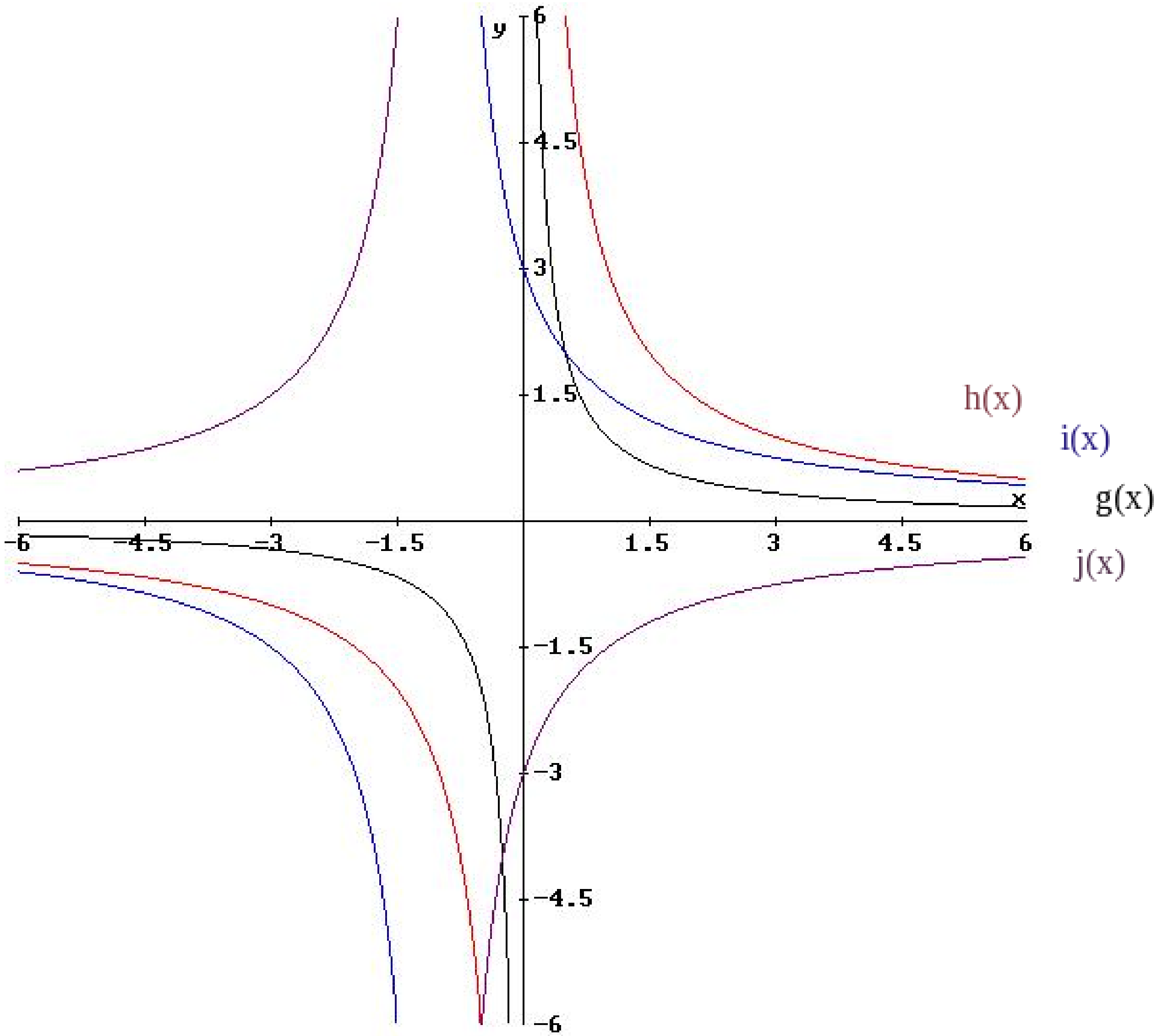
Example Graph: $f(x)=\frac{-3}{x+1}$

base graph: $g(x)=\frac{1}{x}$

vertical scale: $h(x) = 3g(x) = \frac{3}{x}$

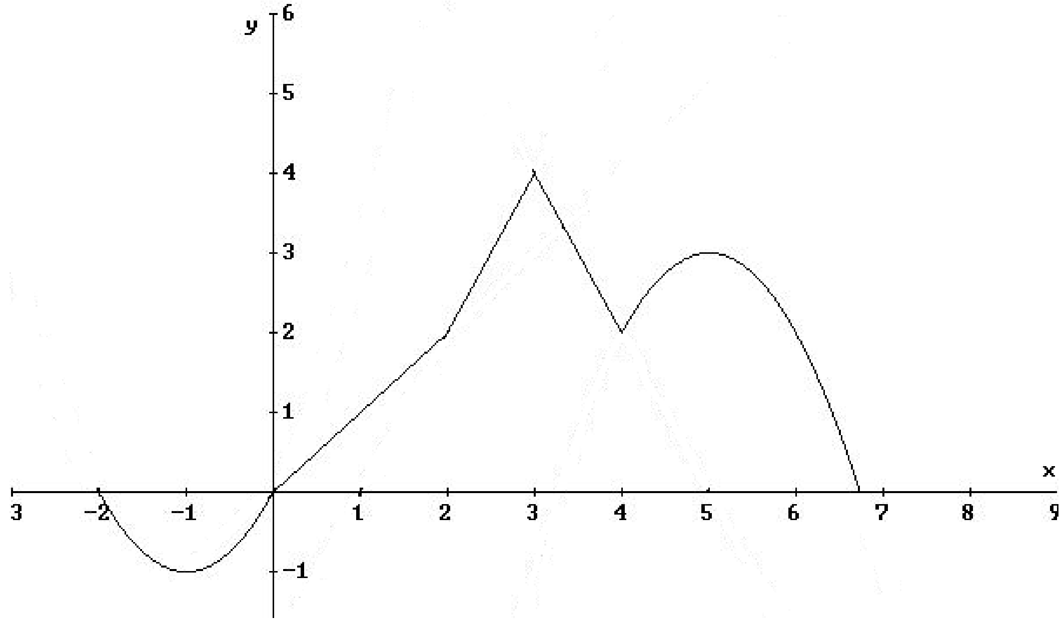
horiz shift: $i(x) = 3g(x+1) = \frac{3}{x+1}$

vertical flip: $j(x) = -3g(x+1) = -\frac{3}{x+1}$



The beauty of this technique is that it works for <u>any</u> function.

Ex *f(x) = ?*



f(x+1) = ?
2f(x) = ?
f(-x) = ?

Effect on Domain

Combinations of Functions	Domain
$(f+g)(x) = f(x)+g(x)$	$D(f) \cap D(g)$
$(f-g)(x) = f(x)-g(x)$	$D(f) \cap D(g)$
Notation: $(f\times g)(x) = f(x)\times g(x)$	$D(f) \cap D(g)$
$(f/g)(x) = \dfrac{f(x)}{g(x)}$	$D(f) \cap D(g),\ g(x)\neq 0$

Example

Let $f(x)=x^2$ D: $(-\infty,\infty)$ $\quad g(x)=\sqrt{x}$ D: $[0,\infty)$

$(f+g)(x)=x^2+\sqrt{x}$ $\quad D: (-\infty,\infty) \cap [0,\infty) \Rightarrow [0,\infty)$

$(f-g)(x)=x^2-\sqrt{x}$ $\quad D: (-\infty,\infty) \cap [0,\infty) \Rightarrow [0,\infty)$

$(f\times g)(x)=x^2\sqrt{x}$ $\quad D: (-\infty,\infty) \cap [0,\infty) \Rightarrow [0,\infty)$

$(f/g)(x)\dfrac{x^2}{\sqrt{x}}$ $\quad D: (-\infty,\infty) \cap [0,\infty)$ but $x\neq 0 \Rightarrow$

$(-\infty,\infty) \cap (0,\infty) \Rightarrow (0,\infty)$

Composite Functions

Notation:

$(f\circ g) = f(g(x))$

$(g\circ f) = g(f(x))$

Be careful with domains --> f(g(x)) has the domain that is all x that gives the right values of g to feed into the function f.

Example:

$f(x)=\sqrt{x-2}$ $\quad$ $g(x)=\frac{1}{x}$

$f(g(x))=\sqrt{\frac{1}{x}-2}$

$f(g(x))=\sqrt{\frac{1-2\mathrm{x}}{x}}$

$g(f(x))=\frac{1}{\sqrt{x-2}}$

Notice that they <u>are</u> not the same!

D: $f(x) \;\Rightarrow\; x-2\geq 0 \;\Rightarrow\; x\geq 2$

$x\in[2,\infty)$

D: $g(x) \;\Rightarrow\; x\neq 0$

$x\in(-\infty,0)\cup(0,\infty)$

D: $f(g(x)) \Rightarrow x\in\left[-\frac{1}{2},\frac{1}{2}\right]$ with $x\neq 0$

$g(x)\geq 2 \Rightarrow \frac{1}{x}\geq 2$

$1\geq 2\mathrm{x}$ or $1\leq 2\mathrm{x}$

$\frac{1}{2}\geq x$ $\quad$ $\frac{1}{2}\leq x$

*Be careful with signs!

D: $g(f(x)) \Rightarrow x\in\mathbb{R}, x\neq 2$

$\cup\; x\in[2,\infty)=(2,\infty)$

$f(x)\neq 0$

$\sqrt{x-2}\neq 0$

$x-2\neq 0 \;\Rightarrow\; x\neq 2$

More on Composite Functions

Essentially when you compose a function, you are making a new function from another function.

The really hard part about composition is understanding what is happening to domain and range.

Specifically, the domain of $f\circ g$ is the set of all *x* in the domain of *g* such that the range of *g(x)* is in the domain of *f*.

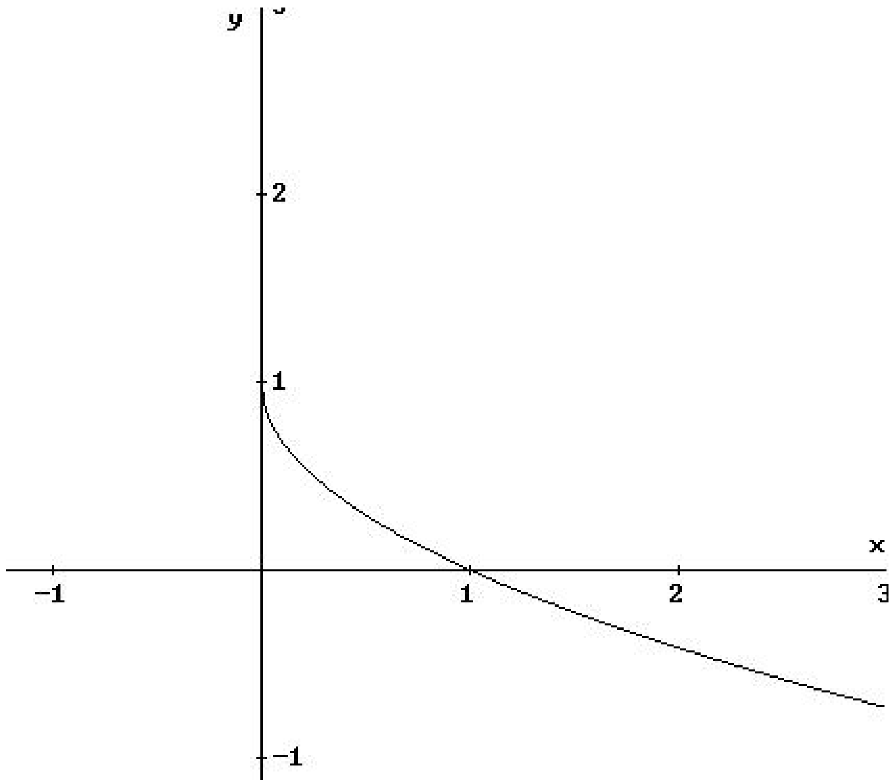
Example: Finding a composition and its domain

Let: $f(x)=\sin(x)$ and $g(x)=1-\sqrt{x}$

To find domain: 1st what is the general domain of *g(x)*?

$g(x)=1-\sqrt{x}$ $\quad$ $x\in[0,\infty)$

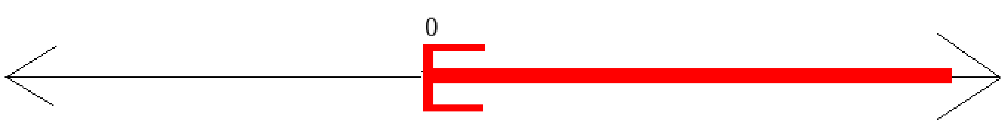
What is the range of *g(x)*?



Graphically, $g(x) \in (-\infty, 1]$

Now, you must ask yourself, what happens when I put those numbers into *f(x) = sin(x)*?
The normal domain of $\sin(x) \Rightarrow x \in (-\infty, \infty)$

Normal Domain of f(x), 0 is not a problem



Domain of g(x), 0 is not allowed

Hence, domain of f(g(x)) = domain of g(x)

2nd find $(g \circ f)(x) = f(f(x)) = 1 - \sqrt{\sin(x)}$

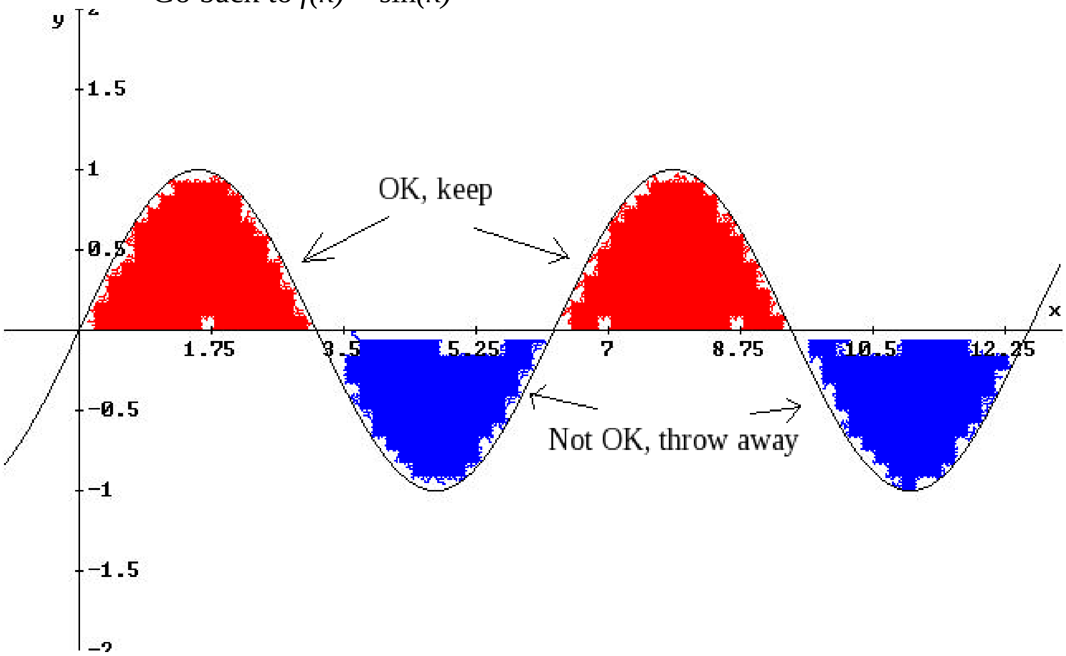
To find domain, what is general domain of f(x)?

f(x) = sin*(x)* $x \in (-\infty, \infty)$

What is the range of f(x)? [-1, 1]

Now, ask yourself, what happens when I put those numbers into
$g(x) = 1 - \sqrt{x}$?
Trouble: the values [-1,1] won't all work. I must restrict them to numbers

between [0, 1].
How?
Go back to *f(x)* = sin*(x)*



So, the original domain of f needs to be restricted to produce any values bigger or equal to 0.

$x \in [0, \pi] \cup [2\pi, 3\pi] \cup [4\pi, 5\pi] \cup \ldots$

This, too, is the domain of $(g \circ f)(x)$. Generally: $x \in [2n\pi, 2n\pi + \pi]$
where *n* is any integer.

$n=0$ $[0, \pi]$
$n=1$ $[2\pi, 3\pi]$
$n=-1$ $[-2\pi, -\pi]$ Note; negative half works too.
$n=2$ $[4\pi, 5\pi]$

Example
Now you try: $(f \circ f)(x)$? where $f(x) = \sin(x)$

Recall: $f(x) = \sin(x)$
$f(f(x)) = \sin(\sin(x))$

NO THAT IS NOT $\sin^2(x)$ **!**

Domain of *f*: $x \in (-\infty, \infty)$
Range of *f*: $f(x) \in [-1, 1]$
Will that work as input to *f(x)*? Yes
So, domain of $f \circ f$: $x \in (-\infty, \infty)$

Again: $(g \circ g)(x)$ where $g(x) = 1 - \sqrt{x}$
$g(g(x)) = 1 - \sqrt{1 - \sqrt{x}}$
Domain of *g*: $x \in [0, \infty)$
Range of *g*: $g(x) \in (-\infty, 1]$
Will that work as input to *g(x)*? No

Only the values from [0, 1] will work.
How to restrict?

$$1-\sqrt{x}=0 \qquad\qquad 1-\sqrt{x}=1$$
$$1=\sqrt{x} \qquad \text{And} \qquad \sqrt{x}=0$$
$$x=1 \qquad\qquad x=0$$

So original domain must be restricted $x \in [0, 1]$