Section 1.1 Functions and Their Representations

Definition A function *f* is a rule that assigns to each element *x* in a set *A* exactly one element, called *f(x)*, in a set *B*.

What does it really mean? It implies a dependence.

Example 1: the perimeter *P* of a square is dependent on what? The lengths of the sides of a square.

P = 4x   when x is the length of a side.

Or, more formally:

P(x) = 4x

Key The notation *P(x)* implies that *P* is dependent on the value of *x*.

Example 2: position of a falling object

$s(t) = \frac{1}{2}gt^2 + v_0t + s_0$

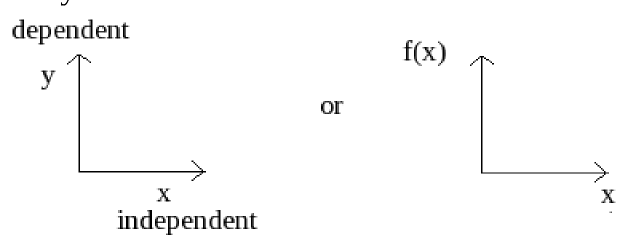
where: g is known (gravity)

v_0 is the initial velocity

s_0 is the initial position

So, *s*, position at any time *t* is dependent on <u>time</u>.

You may have also heard of dependent and independent variables when you graph things.
This is classically represented by:



Language

<u>Domain:</u> (from our definition of a function)

The whole set called *A*

(informally) anything you can put into a function and get a value out

<u>Range:</u>

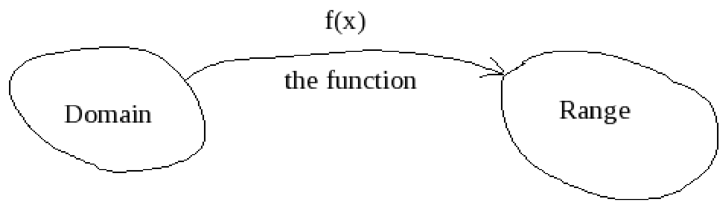
The whole set called *B*

(informally) anything that comes out of a function

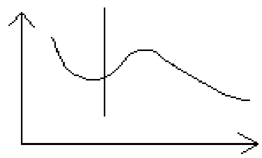
Let's link the language together:

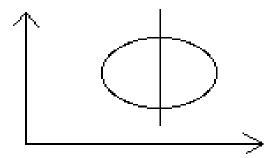
<u>Independent variables</u> are the symbols that represent values that are part of the <u>domain</u> of a function.

<u>Dependent variables</u> are the symbols that represent values that are part of the <u>range</u> of a function.



Vertical Line Test A curve in the *xy* plane is the graph of a function of *x* if and only if no vertical line intersects the curve more than once.

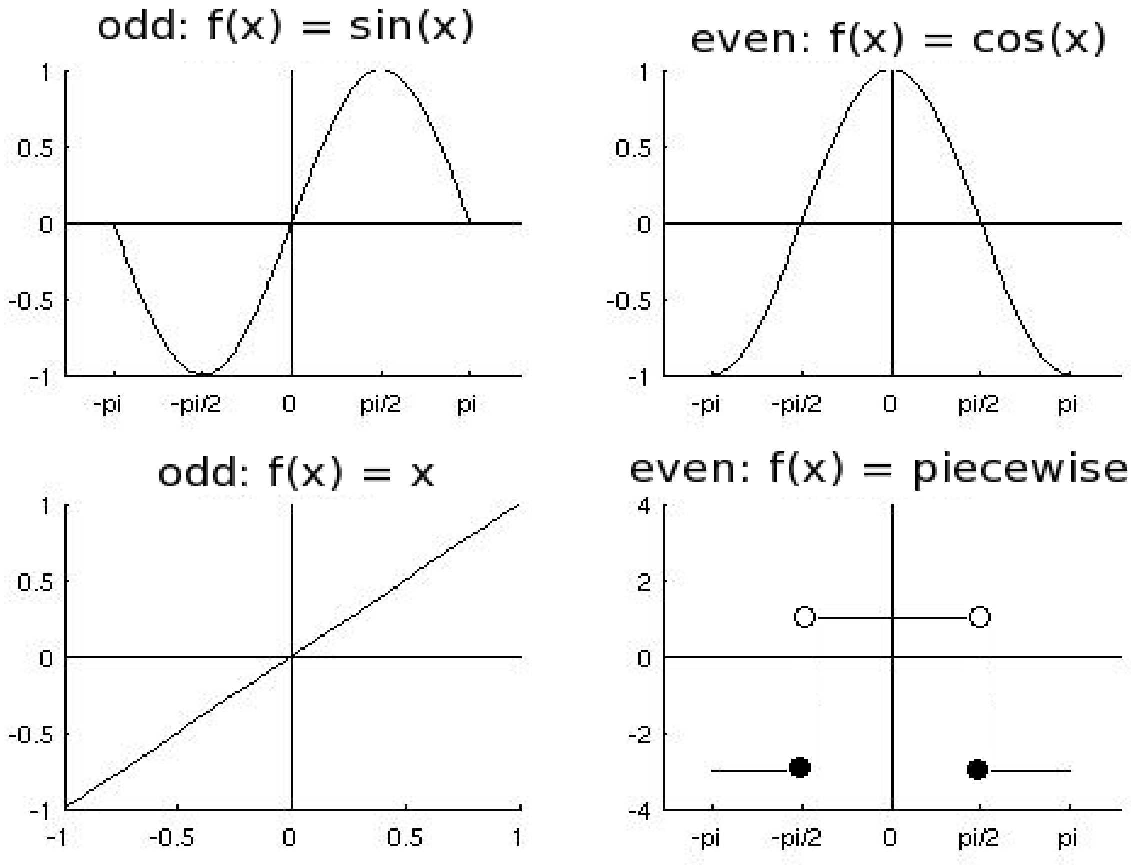




Some examples of functions are polynomials, trigs, logs, absolute value, piecewise, etc.

Symmetry of Functions

Odd	Even
$-f(x) = f(-x)$	$f(x) = f(-x)$
fold along the y then fold along the x to get overlap	fold along the y axis and completely overlaps
symmetric about the origin	symmetric about the y axis

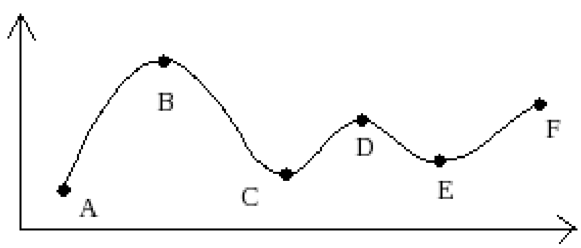


* Neither even nor odd – fails both tests

Increasing / Decreasing Functions

A function *f* is called <u>increasing</u> on an interval *I* if $f(x_1) < f(x_2)$ where $x_1 < x_2$ in *I*
(graphically: f goes up from left to right)

A function *f* is called <u>decreasing</u> on an interval *I* if $f(x_1) > f(x_2)$ where $x_1 < x_2$ in *I*
(graphically: f goes down from left to right)



Increasing: (A,B) & (C,D) & (E,F)
Decreasing: (B,C) & (D,E)

*NOTE: (depicts open vs. [depicts closed

Example Questions

Notation Examples

$f(x) = 5x^3 + 2x^2 - 7$

What is $f(3)$?

$$f(3) = 5(3)^3 + 2(3)^2 - 7$$
$$= 5(27) + 2(9) - 7$$
$$= 135 + 18 - 7$$
$$= 146$$

What is $f(y^2)$?

$$f(y^2) = 5(y^2)^3 + 2(y^2)^2 - 7$$
$$= 5(y^6) + 2(y^4) - 7$$
$$= 5y^6 + 2y^4 - 7$$

What is $f(2y)$?

$$f(2y) = 5(2y)^3 + 2(2y)^2 - 7$$
$$= 5(8y^3) + 2(4y^2) - 7$$
$$= 40y^3 + 8y^2 - 7$$

Example (using difference quotient)

$f(x) = x^2 + x + 1$

What is $\frac{f(x) - f(a)}{x - a}$?

$$f(a) = a^2 + a + 1$$

So: $$\frac{f(x) - f(a)}{x - a} = \frac{(x^2 + x + 1) - (a^2 + a + 1)}{x - a}$$

Finding Domains

Example 1

$$f(x) = \frac{3x}{5x - 2}$$

We know that the denominator cannot equal 0

So: $$5x - 2 \neq 0$$
$$5x \neq 2$$
$$x \neq \frac{2}{5}$$

Are there any other restrictions? What do you think?

Notation: Many different forms for presenting your answer are similar:

$\left\{ x \mid x \in \mathbb{R},\ x \neq \frac{2}{5} \right\}$

or

$x \in (-\infty,\ 2/5) \cup (2/5,\ \infty)$

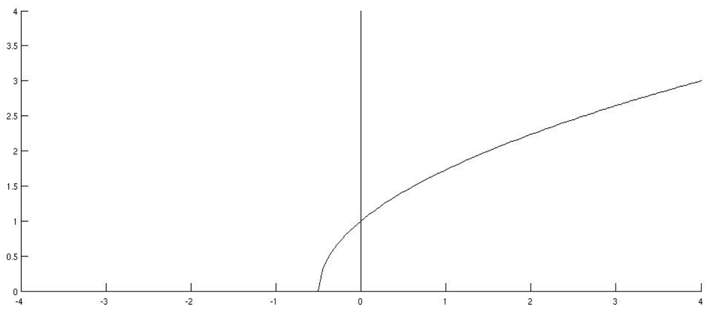
or

$\left\{ x \mid x \neq \frac{2}{5} \right\}$

Example 2

$h(x) = \sqrt{2x+1}$

Sketch:



We know that anything inside the radical must be non-negative.

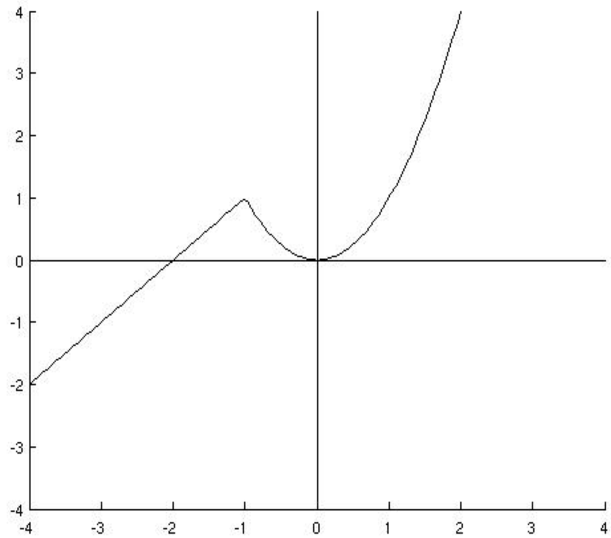
So:
$$2x+1 \geq 0$$
$$2x \geq -1$$
$$x \geq -\frac{1}{2}$$

Domain: $x \in [-1/2, \infty)$ or $\left\{ x \mid -\frac{1}{2} \leq x < \infty \right\}$

Example 3

$$g(x) = \begin{cases} x+2 & if \quad x \leq -1 \\ x^2 & if \quad x > -1 \end{cases}$$

Sketch:



Domain given in notation: $x \in (-\infty, -1] \cup (-1, \infty) \Rightarrow x \in (-\infty, \infty)$
Range: $g(x) \in (-\infty, \infty)$

Manipulating Variables

Example: A rectangle has area 16m^2. Express the perimeter of the rectangle as a function of the length of one of its sides.

What is area? $A = l * w$
$16 = l * w$

And solve for one variable:

$w = \frac{16}{l}$ OR $l = \frac{16}{w}$

What is perimeter?

$$P = 2w + 2l \qquad P = 2w + 2l$$
$$P = 2(\frac{16}{l}) + 2L \qquad P = 2w + 2(\frac{16}{w})$$
$$P = \frac{32}{l} + 2l \quad \text{or} \quad P = 2w + \frac{32}{w}$$
$$P(l) = \frac{32 + 2l^2}{l} \qquad P(w) = \frac{2w^2 + 32}{w}$$

Even / Odd Functions

$$f(x)=\frac{x^2}{x^4+1}$$

Check Even

$$f(-x)=\frac{(-x)^2}{(-x)^4+1}=\frac{x^2}{x^4+1} \quad \text{Even} \quad f(x)=f(-x)$$

Check Odd

$$-f(x)=-\left(\frac{x^2}{x^4+1}\right)=\frac{-x^2}{x^4+1} \quad \text{Not Odd} \quad f(-x)\neq -f(x)$$

Visually, what does this mean? Symmetry over y-axis, because function is EVEN.