Name: ______________________________

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. **[9 parts, 2 points each]** Decide whether the following are true or false. Indicate your answer by writing the entire word. No justification required.

$$A = \{(1,3), \{1,3\}\} \quad B = \{1,3\} \quad C = \{\{3,1\}\} \quad D = \{(3,1)\} \quad E = \{\{\{1,3\}\}\}$$

   (a) $3 \in B$

   (b) $3 \in C$

   (c) $B \in A$

   (d) $B \in C$

   (e) $3 \subseteq B$

   (f) $B \subseteq C$

   (g) $C \subseteq A$

   (h) $A \subseteq D$

   (i) $C \subseteq E$

2. **[2 parts, 3 points each]** Sketch the following sets in the plane.

   (a) $(1,3) \times [-1,1)$

   (b) $\{(x,y) \in \mathbb{R}^2 \colon\ x < 1 \text{ or } y < 1\}$

3. **[6 parts, 3 points each]** Express each set by listing the elements between braces.

   $A = \{\{\}, 2, \{1\}, \{2, 2\}\}$ $B = \{\{1, 1\}, \{2, 3\}, (2, 3), 2\}$ $C = \{\{2\}, \{3, 2\}, \varnothing\}$ $D = \{\varnothing, \{1, 2\}, \{2, 3\}, (3, 2)\}$

   (a) $A \cap B$

   (b) $B \cap C$

   (c) $(B \cup C) - A$

   (d) $(C - A) \times C$

   (e) $\mathcal{P}(C \cap D)$

   (f) $A \cap \mathcal{P}(\mathbb{Z})$

4. **[3 parts, 4 points each]** Give an example of a set with the following properties or explain why no such set exists.

   (a) A set $A \subseteq \mathbb{N}$ such that A and $\overline{A}$ are both infinite.

   (b) A set $B \subseteq \mathbb{Z}$ such that every integer in B is positive and every integer in B is negative.

   (c) A finite set C such that $\mathcal{P}(C)$ is infinite.

5. **[4 parts, 3 points each]** Give Venn Diagrams for each of the following sets relative to a universe U.

   (a) $(A \cup B) \cap C$

   (b) $(A \cup B \cup C) - (A \cap C)$

   (c) $A - (B - C)$

   (d) $\overline{A \cap B} \cup \overline{B \cap C}$

6. **[5 points]** Give two examples of an infinite set A such that $A \in \mathcal{P}(\mathcal{P}(\mathbb{R}))$.

7. **[5 points]** Use Venn Diagrams to decide if the equation $(A - B) - C = A \cap \overline{B} \cap \overline{C}$ is valid for all sets A, B, and C.

8. **[3 parts, 6 points each]** Let $I = \{\alpha \in \mathbb{R}\colon\ \alpha > 0\}$ and let $D_\alpha = \{(x, y) \in \mathbb{R}^2\colon\ y \geq \alpha|x|\}$. Note that $|x|$ is the absolute value of x, so that $|x| = x$ when $x \geq 0$ and $|x| = -x$ when $x < 0$.

   (a) Sketch the example sets $D_{1/2}$, D_1, and D_2.

   (b) Sketch $\bigcap_{\alpha \in I} D_\alpha$.

   (c) Sketch $\bigcup_{\alpha \in I} D_\alpha$.

9. **[6 points]** Express the shaded portion of the following Venn diagram as a set by applying elementary set operations to A, B, and C.