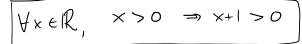
Name: Solutions

Directions: Solve the following problems. Give supporting work/justification where appropriate.

- 1. [3 parts, 1 point each] Translate the following sentences to symbolic logic as directly and simply as possible. Is the statement true or false? Write the entire word.
 - (a) Adding 1 to each positive real number results in a positive real number.





(b) Every real number can be multiplied by some real number to produce a rational number.

YXER, JYER, XYEQ



For all $x \in \mathbb{R}$, we may take y = 0 and $x \cdot 0 = 0 \in \mathbb{Q}$.

(c) The only set which is a subset of every set is the empty set.

 $\forall \text{ sets } A, (\forall \text{ sets } B, A \subseteq B) \Rightarrow A = \emptyset.$

TEUE.

To be a subset of every set requires being a subset of p, and only p subset of p.

2. [3 parts, 1 point each] Translate the following statements/open sentences in symbolic logic to English sentences as simply as possible. Is the statement true or false? Write the entire word.

The number $\sqrt{2}$ is irrational.

TRUE (We've seen this example in class.)

(b) $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}, \exists k \in \mathbb{Z}, x + y = 2k$

There is an integer which, when added to any integer, produces an even integer.

FALSE For all x & Z, either x+0 or x+1 is odd.

(c) $\forall A \subseteq \mathbb{N}, \forall B \subseteq \mathbb{N}, (\exists a \in \mathbb{N}, |A| \le a) \land (\exists b \in \mathbb{N}, |B| \le b) \implies (\exists c \in \mathbb{N}, |A \cup B| \le c)$

The union of two finite sets is finite.

- 3. [2 parts, 2 points each] Negate the following sentences as simply and naturally as possible. (You may translate to and from symbolic logic if helpful, but this is not required.) Is the original statement true or false? Explain.
 - (a) There are at least 7 prime numbers or 5 is less than 0.

Negation. There are fewer than 7 prime numbers and 5 is at least 0.

The original stelement is true | because there are infinitely many primes (so at least 7.)

(b) Every infinite subset of integers contains an infinite subset of even integers.

Origin $\forall A \subseteq \mathbb{Z}$, A infinite $\Rightarrow \exists B \subseteq A$, B infinite $\land B \subseteq \{x \in \mathbb{Z} : x \text{ is even}\}$ Neg: $\exists A \subseteq \mathbb{Z}$, $(A \text{ infinite}) \land \sim (\exists B \subseteq A)$, B infinite $\land B \subseteq \{x \in \mathbb{Z} : x \text{ is even}\}$ $\exists A \subseteq \mathbb{Z}$, $(A \text{ infinite}) \land \forall B \subseteq A$ B finite or $B \not\in \{x \in \mathbb{Z} : x \text{ is even}\}$ $\exists A \subseteq \mathbb{Z}$, $(A \text{ infinite}) \land \forall B \subseteq A$ B finite or B contains a odd integer $\exists A \subseteq \mathbb{Z}$, $(A \text{ infinite}) \land \forall B \subseteq A$ B infinite $\Rightarrow B \text{ contains } a \text{ odd integer}$.

There is an infinite set of integers such that every infinite subset contains an odd integer. The original statement is false. Indeed, the set $\{x \in \mathbb{Z} : x \text{ is odd}\}$ shows that the negation of the statement is true,