Name: Solutions

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. **[6 parts, 1 point each]** We define the following statements and open sentences.

P: 5 is greater than 8. (False) $\quad$ $Q(x)$: x is odd.
$R(x)$: x is negative. $\quad$ $S(A)$: A is a finite set.

Decide whether the following are true or false; indicate your answer by writing the entire word "true" or the entire word "false". Give brief justifications for partial credit.

(a) $\sim P$

Since P is false, $\sim P$ is $\boxed{\text{true}}$.

(b) $\sim Q(4) \wedge \sim P$

$Q(4)$: 4 is odd [false]

$\sim Q(4)$: true

Since both $\sim Q(4)$ and $\sim P$ are true, the statement is $\boxed{\text{true}}$.

(c) $(\sim P \vee S(\mathbb{N})) \wedge (R(-1) \vee Q(8))$ ... TRUE $\wedge$ TRUE

$\sim P$: True

$S(\mathbb{N})$: False "$\mathbb{N}$ is a finite set"

$R(-1)$: True "-1 is negative"

$Q(8)$: False "8 is odd"

So this statement is $\boxed{\text{true}}$.

(d) $P \implies 1 = 2$

FALSE FALSE

Since the hypothesis is false, the implication is $\boxed{\text{true}}$.

(e) $\sim (R(5) \iff Q(6))$

$R(5)$: FALSE "5 is negative"

$Q(6)$: FALSE "6 is odd"

- $R(5) \iff Q(6)$ is <u>true</u> since both $R(5)$ and $Q(6)$ have the same truth value.
- So $\sim(R(5) \iff Q(6))$ is $\boxed{\text{false}}$.

(f) $\sim S(\{1, 2, 4, 8, 16, 32, \dots\}) \iff (R(-1) \implies Q(0))$

- $S(\{1,2,4,\dots\})$: $\{1,2,4,\dots\}$ is a finite set (false), So $\sim S(\{1,2,\dots\})$ is true.
- $R(-1) \implies Q(0)$. Since the hypothesis is true but the conclusion is false, the implication is false.

$R(-1)$: "$-1 < 0$" true

$Q(0)$: 0 is odd false

So, true $\iff$ false is a $\boxed{\text{false}}$ biconditional statement.

2. **[2 parts, 1 point each]** Truth tables and logical equivalence.

   (a) Write a truth table for $(P \iff Q) \implies P$

P	Q	$P \Leftrightarrow Q$	$(P \Leftrightarrow Q) \Rightarrow P$	
T	T	T	T	$(T \Rightarrow T)$
T	F	F	T	$(F \Rightarrow T)$
F	T	F	T	$(F \Rightarrow F)$
F	F	T	F	$(T \Rightarrow F)$

   (b) Give a simple statement which is logically equivalent to $(P \iff Q) \implies P$.

This statement is equivalent to $P \vee Q$.

3. **[2 parts, 1 point each]** Let P, Q, and R be statements. Use the standard logical operands $\sim, \vee, \wedge, \implies, \iff$ to express the following statements.

   (a) P, Q, and R all have the same truth value.

$P \Leftrightarrow Q \Leftrightarrow R$ or $P \Leftrightarrow (Q \Leftrightarrow R)$

or $(P \Leftrightarrow Q) \Leftrightarrow R$

   (b) Q is a necessary condition for P, and R is a sufficient condition for P.

$(P \Rightarrow Q) \wedge (R \Rightarrow P)$

Note: $(R \Rightarrow P) \Rightarrow Q$ and $R \Rightarrow (P \Rightarrow Q)$ are not equivalent, since both hold when R is true and P is false.