Name: Solutions

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. **[2 parts, 2 points each]** Determine the following sets.

(a) $\bigcup_{n=1}^{\infty}\left[\frac{1}{n},1\right] = [1,1] \cup [\frac{1}{2},1] \cup [\frac{1}{3},1] \cup [\frac{1}{4},1] \cup \cdots$

$= \{x \in \mathbb{R} : 0 < x \leq 1\}$

$= (0,1]$

(b) $\bigcap_{n=1}^{\infty}\left[\frac{1}{n},1\right] = [1,1] \cap [\frac{1}{2},1] \cap [\frac{1}{3},1] \cap \cdots$

$= [1,1]$ (since all other intervals contain $[1,1]$)

$= \{1\}$

2. **[6 parts, 1 point each]** Determine whether or not the following are statements. In the case of a statement, say if it is true or false, if possible. Briefly explain your reasoning.

(a) $+8 + \times\mathbb{R}^2$

Not a statement : Nonsense

(b) For all real numbers x and y, if $xy = 0$ then $x = 0$ or $y = 0$.

TRUE Statement

Suppose $xy = 0$. If $x = 0$, then the statement is true. Otherwise $x \neq 0$ and we can divide both sides of $xy = 0$ by x to get $y = 0$.

(c) The sum of two prime numbers cannot be prime.

FALSE Statement

Note that $2+3=5$, and 2, 3, and 5 are all prime.

(d) If 3 plus 4, then 7.

Not a Statement

"3 plus 4" and "7" are expressions, not statements. In "If P, then Q", both P and Q should be statements.

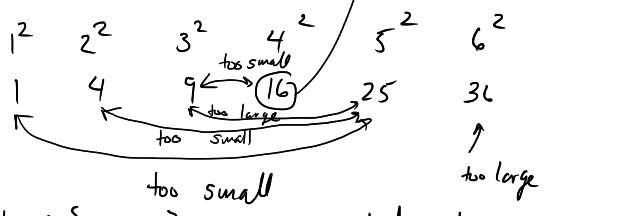
(e) The best color is purple.

Not a statement.

Matter of opinion

(f) There are integers a and b such that $a^2 + b^2 = 30$.

FALSE Statement



By checking a^2+b^2 for $a, b \in \{1, \ldots, 5\}$, we see that there are no such integers a and b.