Name: _____

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. **[3 points]** Let $a \in \mathbb{R}$. Prove that if a^3 is irrational, then a is irrational.

2. **[3 points]** Let $a, b \in \mathbb{Z}$ with $b > 0$. Prove that there is at most one pair of integers (q, r) such that $a = bq + r$ and $0 \leq r < b$.

3. Let $a, b, c, d \in \mathbb{R}$, let $f(x) = ax + b$, and let $g(x) = cx + d$.

   (a) **[3 points]** Show that there exists $x \in \mathbb{R}$ such that $f(x) = g(x)$ if and only if $a \neq c$ or $d = b$.

   (b) **[1 point]** Fill in the blank to make the following statement true: There exists a unique $x \in \mathbb{R}$ such that $f(x) = g(x)$ if and only if _____.