Math303 | **Quiz 1** | **Mon. Jan. 22, 2024**

Name: Solutions

Directions: Solve the following problems. Give supporting work/justification where appropriate.

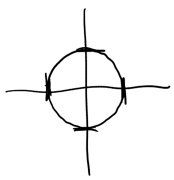
1. **[2 parts, 1 point each]** Express the following sets using a list between braces, using the ellipses if necessary.

   (a) $\{3n-1\colon\ n \in \mathbb{Z} \text{ and } |n| \le 3\}$

   $\boxed{\{-10, -7, -4, -1, 2, 5, 8\}}$

   $n=-3$ $n=3$

   (b) $\{(x,y)\colon\ x, y \in \mathbb{Z} \text{ and } x^2 + y^2 = 1\}$

   $= \boxed{\{(1,0), (0,1), (-1,0), (0,-1)\}}$

2. **[4 parts, 1 point each]** Determine whether the following sets are infinite or finite. If the set is finite, then determine its cardinality.

   (a) $\{1, \{1\}, \{\{1\}\}, \{\{\{1\}\}\}, \ldots\}$

   $\boxed{\text{Infinite}}$

   (b) $\{\mathbb{R}\}$

   Since $\{\mathbb{R}\}$ is the singleton set whose only member is the set $\mathbb{R}$ of real numbers, $|\{\mathbb{R}\}| = \boxed{1}$.

   (c) $\{x \in \mathbb{R}\colon\ x^2 = 1\}$

   $$x^2 = 1 \iff x^2 - 1 = 0$$
   $$\iff (x-1)(x+1) = 0$$
   $$x = 1,\ x = -1$$

   So the set equals $\{-1, 1\}$ and has size $\boxed{2}$.

   (d) $\{1, 2, 3, \{1,2\}, 1, \{2,1,2\}\}$

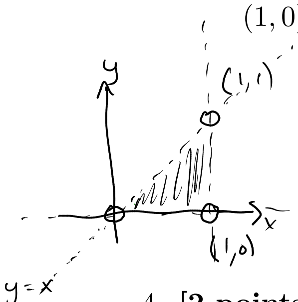
   $=$ $=$

   This set contains 3 integers and one set ($\{1,2\} = \{2,1,2\}$), and so it has size $\boxed{4}$.

3. **[2 parts, 1 point each]** Use set-builder notation to express the following sets compactly.

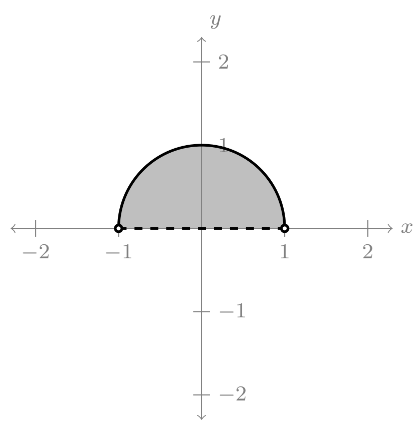
   (a) $\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots\}$ $=$ $\boxed{\left\{ \frac{n-1}{n} : n \in \mathbb{Z} \text{ and } n \geq 2 \right\}}$

   (b) The set of all points (x, y) in the interior of the triangle with vertices $(0,0)$, $(1,1)$, and $(1,0)$.



$\boxed{\{(x,y) \in \mathbb{R}^2 : 0 < x < 1 \text{ and } 0 < y < x\}}$

4. **[2 points]** Use set-builder notation to express the subset of $\mathbb{R}^2$ displayed below.



$\boxed{\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1 \text{ and } y > 0\}}$