Directions: You may work to solve these problems in groups, but all written work must be your own. Unless the problem indicates otherwise, all problems require some justification; a correct answer without supporting reasoning is not sufficient. Submissions must be stapled. See "Guidelines and advice" on the course webpage for more information.

1. Let $A = \{1, 2, 3\}$ and $B = \{\sin, \cos\}$. List the elements of the following sets.

   (a) $\mathcal{P}(B)$

   (b) $\mathcal{P}(B \times \{a\})$

2. Suppose that $|A| = m$ and $|B| = n$. Find the given cardinalities.

   (a) $|\mathcal{P}(\mathcal{P}(A))|$

   (b) $|\mathcal{P}(A \times \mathcal{P}(B))|$

   (c) $|\mathcal{P}(A) \times \mathcal{P}(B)|$

   (d) $|\{X \subseteq \mathcal{P}(A) \colon |X| \leq 1\}|$

3. Suppose $A = \{b, c, d\}$ and $B = \{a, b\}$. Find the following sets.

   (a) $A \cup B$

   (b) $A - B$

   (c) $(A \times B) \cap (B \times B)$

   (d) $(A \times B) \cap B$

   (e) $\mathcal{P}(A) - \mathcal{P}(B)$

   (f) $\mathcal{P}(A) \cap \mathcal{P}(B)$

   (g) $\mathcal{P}(A \cap B)$

4. [BP 1.5.6] Sketch the sets $X = [-1, 3] \times [0, 2]$ and $Y = [0, 3] \times [1, 4]$ on the plane $\mathbb{R}^2$. On separate drawings, shade in the sets $X \cup Y$, $X \cap Y$, $X - Y$, and $Y - X$.

5. [BP 1.5.10] Is the statement $(\mathbb{R} - \mathbb{Z}) \times \mathbb{N} = (\mathbb{R} \times \mathbb{N}) - (\mathbb{Z} \times \mathbb{N})$ true or false? Justify your answer.

6. Let the universe U be $\{1, 2, 3, \ldots, 8\}$, let $A = \{1, 2, 3, 4, 5\}$, and let $B = \{3, 4, 5, 6, 7\}$. Find the following sets.

   (a) $\overline{A}$

   (b) $\overline{B}$

   (c) $A \cap \overline{A}$

   (d) $A \cap \overline{B}$

   (e) $A \cup \overline{A}$

   (f) $\overline{A \cup B}$

   (g) $\overline{A} \cap \overline{B}$

   (h) $\overline{A \cap B}$

   (i) $\overline{A} \cup \overline{B}$

7. [BP 1.6.{5,6}] Sketch the following sets in the plane $\mathbb{R}^2$. In a separate drawing, shade in the complement.

   (a) $\{(x, y) \in \mathbb{R}^2 \colon 1 \leq x^2 + y^2 \leq 4\}$

   (b) $\{(x, y) \in \mathbb{R}^2 \colon y < x^2\}$

8. [BP 1.7.8] Suppose sets A and B are in a universal set U. Draw Venn diagrams for $\overline{A \cup B}$ and $\overline{A} \cap \overline{B}$. Based on your drawings, do you think it is true that these sets are equal? (Compare with 4(f) and 4(g).)

9. A team of 2 pirates, Blackbeard and Israel Hands, steals 500 gold coins. They have an unusual system of distributing the coins. They arrange all the coins in a row, from coin 1 to coin 500, and ensure that each coin has it's heads side up. On the first day, they flip over each coin. On the second day, they flip over every other coin, starting with coin 2, then coin 4, then coin 6, and so forth. On the third day, they flip over every third coin, starting with coin 3, then coin 6, then coin 9, and so forth. They continue this pattern until day 500, when they flip over only coin 500. Blackbeard keeps all coins that have heads side facing up, and Israel Hands keeps the rest. How many coins does each pirate keep? Why?