Name: Solutions

Directions: Show all work. No credit for answers without work.

1. [**5 points**] Solve the following system of congruences; your solution should identify the set of all possible solutions.

$$x \equiv 12 \pmod{15} \qquad 5x \equiv 10 \pmod{19}$$

$15 = 3 \cdot 5$, 19 prime $\Rightarrow$ moduli are rel prime

$4 \cdot 5x \equiv 10 \cdot 4 \pmod{19}$

$x \equiv 40 \equiv 2 \pmod{19}$.

$M = 15 \cdot 19 = 15(20-1)$
$= 300 - 15$
$= 285$

i	m_i	$z_i = M/m_i$	$z_i \pmod{m_i}$	$y_i = z_i^{-1} \pmod{m_i}$	x_i
1	15	19	4	4	12
2	19	15	15	-5	2

$19 = (1)(15) + 4$
$15 = (3)(4) + 3$
$4 = (1)(3) + 1$

$1 = 4 - (1)(3)$
$= (4)(4) + (-1)(15)$
$= (4)(19 + (-1)(15)) + (-1)(15)$
$= (4)(19) + (-5)(15)$

So $x = 19 \cdot 4 \cdot 12 + 15 \cdot (-5) \cdot (2) \pmod{M}$
$= (20-1) \cdot 48 - 150 \pmod{285}$
$= 960 - 48 - 150 \pmod{285}$
$= 760 + 2 = 762 \pmod{285}$
$\equiv 762 - 570 \pmod{285}$
$\equiv 192 \pmod{285}$.

$285 \cdot 2 = 570$

So set of solns is $\boxed{\{192 + k \cdot 285 : k \in \mathbb{Z}\}}$

2. [**5 points**] A large box containing n jellybeans sits on the teacher's desk; we know $n \leq 10,000$. When the jellybeans are divided among 31 students, 17 are left over. When the jellybeans are divided among 17 students, 10 are left over. When the jellybeans are divided among 23 students, 9 are left over. Find n.

$n \equiv 17 \pmod{31}$ $\quad n \equiv 10 \pmod{17}$ $\quad n \equiv 9 \pmod{23}$; all moduli prime

$M = 31 \cdot 17 \cdot 23 = 12121$

i	m_i	$z_i = M/m_i$	$z_i \bmod m_i$	$y_i = z_i^{-1} \pmod{m_i}$	x_i
1	31	391	19	-13	17
2	17	713	$16 = -1$	-1	10
3	23	527	$21 = -2$	-12	9

$x = (391)(-13)(17)$
$+ (713)(-1)(10)$
$+ (527)(-12)(9) \pmod{M}$
$= -86411 - 7130 - 56916$
$= -150,457$
$\equiv 7116 \pmod{M}$

So $\boxed{n = 7116}$.

$31 = (1)(19) + 12$
$19 = (1)(12) + 7$
$12 = (1)(7) + 5$
$7 = (1)(5) + 2$
$5 = (2)(2) + 1$

$1 = 5 - (2)(2)$
$= 5 - (2)[7 - (1)(5)]$
$= (3)(5) - (2)(7)$
$= (3)[12 - (1)(7)] - (2)(7)$
$= (3)(12) + (-5)(7)$
$= (3)(12) + (-5)[19 - (1)(12)]$
$= (8)(12) + (-5)(19)$
$= (8)[31 - (1)(19)] + (-5)(19)$
$= (8)(31) + (-13)(19)$