Name: Solutions

Directions: Show all work. No credit for answers without work.

1. **[3 parts, 1 point each]** Simplify the following expressions if possible.

(a) $\dfrac{(x^2 \cdot x^3)^5}{x^6 + x^7}$

$$= \frac{(x^{2+3})^5}{x^6(1+x)}$$

$$= \frac{(x^5)^5}{x^6(1+x)}$$

$$= \frac{x^{25}}{x^6(1+x)} = \boxed{\frac{x^{19}}{1+x}}$$

(b) $\sqrt{x^2 + y^2}$

$\boxed{\text{No simplification possible}}$

Note: $\sqrt{A+B} \neq \sqrt{A} + \sqrt{B}$

(c) $\dfrac{2x + 15}{x + 5}$

OK: $\boxed{\text{No simplification possible}}$

Also OK: $\dfrac{2x+15}{x+5} = \dfrac{2(x+5)+5}{x+5}$

$$= \frac{2\cancel{(x+5)}}{\cancel{x+5}} + \frac{5}{x+5} = \boxed{2 + \frac{5}{x+5}}$$

2. **[2 parts, 1 point each]** Find the derivatives of the following functions.

(a) $f(x) = \ln(e^x + \ln(x))$

By Chain Rule

$$f'(x) = \frac{d}{dx}\left[\ln(e^x + \ln(x))\right] = \frac{1}{e^x + \ln(x)} \cdot \frac{d}{dx}\left[e^x + \ln(x)\right]$$

$$= \frac{1}{e^x + \ln(x)} \cdot \left(e^x + \frac{1}{x}\right)$$

$$= \frac{e^x + \frac{1}{x}}{e^x + \ln(x)} = \frac{x\left(e^x + \frac{1}{x}\right)}{x(e^x + \ln(x))} = \boxed{\frac{xe^x + 1}{x(e^x + \ln(x))}}$$

(b) $g(x) = x^{\sin(x)}$

Use implicit differentiation:

$$\ln g(x) = \ln\left[x^{\sin(x)}\right]$$

$$\ln(g(x)) = (\sin(x)) \cdot \ln(x)$$

$$\frac{d}{dx}\left[\ln(g(x))\right] = \frac{d}{dx}\left[\sin(x) \cdot \ln(x)\right]$$

$$\frac{1}{g(x)} \cdot g'(x) = (\cos x)(\ln(x)) + \sin(x) \cdot \frac{1}{x}$$

So $g'(x) = g(x) \cdot \left(\cos(x)\ln(x) + \dfrac{\sin x}{x}\right)$

$$= \boxed{x^{\sin(x)}\left(\cos(x)\ln(x) + \frac{\sin x}{x}\right)}$$

3. **[2 points]** The function f takes an array of integers as input. Let $B = [2, 5, 3, 6]$; here array indexing starts with 1, so that $B[1] = 2$ and $B[4] = 1$. What does f return when called with input B? Explain your solution for partial credit.

```
f(A[1..n]):
  s ← 0
  for i = 1 to n:
    if i is even:
      s ← s + i · A[i]
    else:
      s ← s − i · A[i]
  return s
```

s=0

i=1 ← odd	i=2 ← even	i=3 ← odd	i=4 ← even	So f(B)
s← s−i·A[i]	s← s+i·A[i]	s← s−i·A[i]	s← s+i·A[i]	returns $\boxed{23}$.
s← 0−1·2	s← (−2)+(2)·5	s← 8−3·3	s← −1+4·6	
s← −2	s← 8	s← −1	s← 23	

4. **[3 points]** Given an array $A[1..n]$ of distinct integers in **sorted order** and an integer x, the function `find`$(A[1..n], x)$ should return `True` if x is one of the values in A and `False` otherwise. Give a pseudocode implementation of `find`$(A[1..n], x)$. A correct implementation is worth 2 points; a correct, *efficient* implementation is worth 3 points.

Correct but inefficient O(n) algorithm:

```
find (A[1..n], x):
    for i=1 to n:
        if A[i] = x:
            return True
    return False
```

Correct, efficient O(log n) algorithm – Binary Search

floor / round down

```
find (A[1..n], x):
    lb ← 0, ub ← n+1
    while (ub−lb) ≥ 2:
        mid ← lb + ⌊(ub−lb)/2⌋
        if A[mid] = x:
            return True
        else if A[mid] < x:
            lb ← mid
        else:                      // A[mid] > x
            ub ← mid
    return False
```