Math373 **Homework 1** **Due Wed, Jan. 18, 2023**

Directions: Solve the following problems. All written work must be your own. See the course syllabus for detailed rules.

1. The Caesar cipher.

   (a) Encrypt the message “exchange all assets” using a Caesar cipher with a forward shift of 5 characters.

   (b) Decrypt the following message, which has been encoded with a Caesar cipher.

   `DPYLA OLTVU LFAVT VYYVD`

2. [JJJ 1.{9,10}.c] **Note: This problem is moved to HW2.** Let $d = \gcd(16261, 85652)$. Use the extended Euclidean algorithm to find integers u and v such that $16261u + 85652v = d$.

3. *Practice with large numbers.* Define a sequence of numbers $a_0, a_1, a_2, \ldots$ recursively by $a_0 = a_1 = a_2 = 1$ and $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for $n \geq 3$. For example, $a_3 = a_2 + a_1 + a_0 = 1 + 1 + 1 = 3$ and $a_4 = a_3 + a_2 + a_1 = 3 + 1 + 1 = 5$. Also, $a_{10} = 193$ and $a_{20} = 85525$.

   (a) Consider the following recursive algorithm for computing a_n.

   ```
   $A(n)$:
      if $n \leq 2$ then
           return 1

      return $A(n-1) + A(n-2) + A(n-3)$
   ```

   Comment on the efficiency of this code. What is the run-time of this algorithm?

   (b) Give a more efficient algorithm to compute a_n.

   (c) What are a_{16} and a_{55}?

   (d) Define a new sequence b_n such that b_n is the sum of the digits in a_n. For example, since $a_{10} = 193$, we have that $b_{10} = 1 + 9 + 3 = 13$ and since $a_{20} = 85525$, we have that $b_{20} = 8 + 5 + 5 + 2 + 5 = 25$. What is b_{20000}? Note: `fib.py` contains a function `digit_sum(n)` that computes the sum of the digits in n.