

Name: _____

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. **[3 parts, 1 point each]** Translate the following sentences to symbolic logic as directly and simply as possible. Is the statement true or false? Write the entire word.

(a) The emptyset is a subset of every set.

(b) There is an integer which is both a perfect square and a perfect cube.

(c) Each rational number can be bounded above by some integer.

2. **[3 parts, 1 point each]** Translate the following statements/open sentences in symbolic logic to English sentences as simply as possible. Is the statement true or false? Write the entire word.

(a) $\sim (\exists x \in \mathbb{Q}, x^2 = 2)$

(b) $\forall S \subseteq \mathbb{R}, [(\exists a \in \mathbb{N}, |S| \leq a) \implies (\exists m \in S, \forall x \in S, x \leq m)]$

(c) $\forall x \in \mathbb{R}, \forall \varepsilon \in \mathbb{R}, [\varepsilon > 0 \implies \exists y \in \mathbb{Q}, |x - y| < \varepsilon]$

3. [2 parts, 2 points each] Negate the following sentences as simply and naturally as possible. (You may translate to and from symbolic logic if helpful, but this is not required.) Is the **original statement** true or false? Explain.

(a) Every pair of distinct lines in the plane intersects at a unique point.

(b) There is an infinite set of integers such that no integer in the set divides some other integer in the set.