Name: _____

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. **[6 parts, 1 point each]** We define the following statements and open sentences.

$$P\colon\ 5 \text{ is even.} \qquad Q(x)\colon\ x \text{ is odd.}$$
$$R(x)\colon\ x \text{ is negative.} \qquad S(A)\colon\ A \text{ is a finite set.}$$

Decide whether the following are true or false; indicate your answer by writing the entire word "true" or the entire word "false". Give brief justifications for partial credit.

(a) $\sim P$

(b) $Q(3) \vee \sim P$

(c) $(P \vee S(\mathbb{N})) \wedge (R(-1) \vee Q(5))$

(d) $P \implies S(\mathbb{R})$

(e) $\sim (R(5) \iff Q(6))$

(f) $\sim S(\varnothing) \iff (R(-1) \implies Q(0))$

2. **[2 parts, 1 point each]** Truth tables and logical equivalence.

   (a) Write a truth table for $(P \implies Q) \implies P$

   (b) Give a simple statement which is logically equivalent to $(P \implies Q) \implies P$.

3. **[2 parts, 1 point each]** Let P, Q, and R be statements. Use the standard logical operands $\sim, \vee, \wedge, \implies, \iff$ to express the following statements.

   (a) At least one statement in $\{P, Q, R\}$ is true.

   (b) Having exactly one of $\{Q, R\}$ hold is a necessary condition for P.