Name: __Solutions__

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. [**6 parts, 1 point each**] We define the following statements and open sentences.

$$P\colon 5 \text{ is even.} \qquad\qquad Q(x)\colon x \text{ is odd.}$$
$$R(x)\colon x \text{ is negative.} \qquad\qquad S(A)\colon A \text{ is a finite set.}$$

Decide whether the following are true or false; indicate your answer by writing the entire word "true" or the entire word "false". Give brief justifications for partial credit.

(a) $\sim P$: " 5 is odd ".

This is [true], since 5 is an odd integer.

(b) $Q(3) \vee \sim P$ " $\big(3$ is odd$\big)$ or $(5$ is odd$)$ ",

This is [true] since at least one of the integers in $\{3,5\}$ is odd.

(c) $(P \vee S(\mathbb{N})) \wedge (R(-1) \vee Q(5))$: $\underset{\text{FALSE}}{(5 \text{ is even}} \text{ OR } \underset{\text{FALSE}}{\mathbb{N} \text{ is finite})} \text{ AND } (\underset{\text{TRUE}}{-1 \text{ is neg}} \text{ or } \underset{\text{TRUE}}{5 \text{ is odd}})$

This is [False] since P and $S(\mathbb{N})$ both fail.

(d) $P \implies S(\mathbb{R})$ FALSE $\implies$ FALSE

This is [true], since the hypothesis P fails.

(e) $\sim (R(5) \iff Q(6))$ Not $\big(\underset{\text{FALSE}}{5 \text{ is negative}} \iff \underset{\text{FALSE}}{6 \text{ is odd}} \big)$

This is [false], since both $R(5)$ and $Q(6)$ have the same truth value.

(f) $\sim S(\varnothing) \iff (R(-1) \implies Q(0))$: $\big(\sim \varnothing \text{ is finite}\big) \iff \big(\underset{}{-1 \text{ is neg}} \implies 0 \text{ is odd}\big)$

           FALSE $\iff$ (TRUE $\implies$ FALSE)

           FALSE $\iff$ FALSE

               TRUE

This statement is [true] since both $\sim S(\varnothing)$ and $R(-1) \implies Q(0)$ are false.

2. [**2 parts, 1 point each**] Truth tables and logical equivalence.

   (a) Write a truth table for $(P \implies Q) \implies P$

P	Q	$P \Rightarrow Q$	$(P \Rightarrow Q) \Rightarrow P$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	F

   (b) Give a simple statement which is logically equivalent to $(P \implies Q) \implies P$.

   From the table above, $(P \Rightarrow Q) \Rightarrow P$ is equivalent to $\boxed{P}$.

3. [**2 parts, 1 point each**] Let P, Q, and R be statements. Use the standard logical operands $\sim, \vee, \wedge, \implies, \iff$ to express the following statements.

   (a) At least one statement in $\{P, Q, R\}$ is true.

   $$\boxed{P \vee Q \vee R}$$

   (b) Having exactly one of $\{Q, R\}$ hold is a necessary condition for P.

   "If P holds, then exactly one of $\{Q, R\}$ hold".

   $$\boxed{P \Rightarrow \sim(Q \iff R)}$$