Name: Solutions

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. **[1 parts, 10 points each]** Decide whether or not the following are statements. In the case of a statement, say if it is true or false, if possible. Briefly explain your reasoning.

   (a) $0 \cdot 5 = \varnothing$

   FALSE Statement — LHS is an integer, RHS is a set.

   (b) An even integer plus an odd integer equals an odd integer.

   TRUE Statement — $\underbrace{2m}_{\text{even}} + \underbrace{2t+1}_{\text{odd}} = \underbrace{2(m+t)+1}_{\text{odd}}$

   (c) Always $\mathcal{P}(A)$ when A is a set.

   Not a Statement — $\mathcal{P}(A)$ is a set, not a true/false claim.

   (d) If a, b, and c are integers and $ab = ac$, then $b = c$.

   FALSE Statement — If $a = 0$, then b and c could be any integers

   (e) Every set is finite or infinite.

   True Statement — By definition, each set is either finite or infinite

   (f) $1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \cdots = \frac{3}{2}$.

   True Statement — This is a geom. series $1 + x + x^2 + \cdots = \frac{1}{1-x}$, $|x|<1$, with $x = \frac{1}{3}$. So series equals $\frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$.

   (g) If x is an integer, then $x < 4$ or $x > 4$.

   FALSE Statement — If $x = 4$ then both $4 < 4$ and $4 > 4$ fail.

   (h) $(\mathbb{Z} \cup \mathbb{N})$ or $(\mathbb{N} \cup \mathbb{Z})$

   Not a Statement: $\mathbb{Z} \cup \mathbb{N}$ and $\mathbb{N} \cup \mathbb{Z}$ are sets, not true/false claims

   (i) $\mathbb{Z} \cup \mathbb{N} \subseteq \mathbb{Q}$

   True Statement — Every natural number and each integer is a rational number

   (j) If A and B are sets, then $|A| - |B| \geq |A - B|$.

   FALSE Statement — For example, if $A = \{1, 2\}$ and $B = \{2, 3\}$, then $|A| - |B| = 2 - 2 = 0$ but $|A - B| = |\{1\}| = 1$ and $0 \not\geq 1$.