Name: Solutions

Directions: Solve the following problems. Give supporting work/justification where appropriate.

1. **[2 parts, 1 point each]** Express the following sets using a list between braces, using the ellipses if necessary.

   (a) $\{2^n\colon\ n \in \mathbb{Z} \text{ and } |n| \le 3\}$

   $\{2^{-3}, 2^{-2}, 2^{-1}, 2^0, 2^1, 2^2, 2^3\} = \boxed{\{\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, 1, 2, 4, 8\}}$

   (b) $\{(x, y)\colon\ x, y \in \mathbb{Z} \text{ and } xy = 25\}$

   $\boxed{\{(1,25), (5,5), (25,1), (-1,-25), (-5,-5), (-25,-1)\}}$

2. **[4 parts, 1 point each]** Determine whether the following sets are infinite or finite. If the set is finite, then determine its cardinality.

   (a) $\{1, \{1\}, \{\{1\}\}, \{\{\{1\}\}\}, \ldots\}$

   $\boxed{\text{Infinite}}$

   (b) $\{\{1, 2\}, \{2, 1\}, \mathbb{R}\}$

   $\{1,2\} = \{2,1\}$, so this is a set that contains $\boxed{2}$ elements: a set of size 2 and the set of real numbers.

   (c) $\{x \in \mathbb{R}\colon\ x^2 = 1\}$

   $x^2 = 1 \iff x^2 - 1 = 0$
   $\iff (x-1)(x+1) = 0$
   $x = 1,\ x = -1$

   So the set equals $\{-1, 1\}$ and has size $\boxed{2}$.

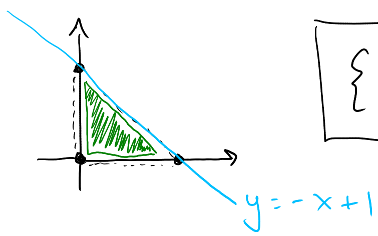
   (d) $\{\varnothing, \{\}, \{x \in \mathbb{Q}\colon\ x \text{ is not an integer}\}\}$

   $\varnothing = \{\}$, so this set has size $\boxed{2}$.

3. **[2 parts, 1 point each]** Use set-builder notation to express the following sets compactly.

   (a) $\{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots\}$ $=$ $\boxed{\left\{ \frac{1}{n} : n \in \mathbb{Z} \text{ and } n \geq 2 \right\}}$

   (b) The set of all points (x, y) in the interior of the triangle with vertices $(0, 0)$, $(0, 1)$, and $(1, 0)$.

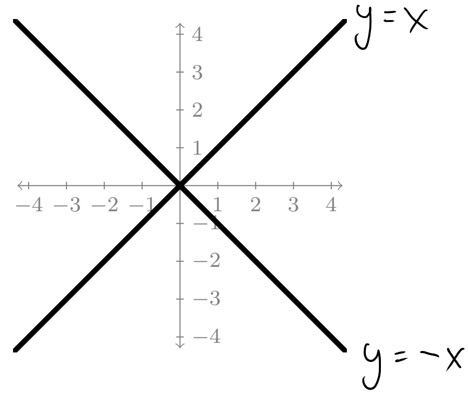


$$\boxed{\{ (x,y) \in \mathbb{R}^2 : x > 0 \text{ and } y > 0 \text{ and } x+y < 1 \}}$$

4. **[1 point]** Is there a set A which satisfies the following conditions: (1) every element in A is an even integer, and (2) every element in A is an odd integer? If so, then give an example of such a set. If not, then explain why not.

$\boxed{\text{Yes, the empty set}}$ has both of these properties.

5. **[1 point]** Use set-builder notation to express the subset of $\mathbb{R}^2$ displayed below.



$$\boxed{\{ (x,y) \in \mathbb{R}^2 : |x| = |y| \}}$$