Directions: You may work to solve these problems in groups, but all written work must be your own. Unless the problem indicates otherwise, all problems require some justification; a correct answer without supporting reasoning is not sufficient. Submissions must be stapled. See "Guidelines and advice" on the course webpage for more information.

1. Let $A = \{1, 2, 3\}$ and $B = \{\sin, \cos\}$. List the elements of the following sets.

   (a) $B \times A$
   (b) $B \times (A \times B)$
   (c) $B \times A \times \emptyset$
   (d) $A \times \{\emptyset\}$
   (e) $\mathcal{P}(B)$
   (f) $\mathcal{P}(B \times \{a\})$

2. List the subsets of the following sets.

   (a) $\{\mathbb{R}, \mathbb{N}, \mathbb{Q}\}$
   (b) $\emptyset$
   (c) $\{\{\mathbb{N}\}\}$

3. Express the set $\{X \subseteq \mathbb{N} \colon |X| \leq 1\}$ by listing its elements between braces, using ellipses if necessary.

4. Decide whether the following statements are true or false. Give explanations.

   (a) $\mathbb{R}^2 \subseteq \mathbb{R}^3$
   (b) $\{(x, y) \in \mathbb{R}^2 \colon x^2 - x = 0\} \subseteq \{(x, y) \in \mathbb{R}^2 \colon x - 1 = 0\}$

5. Suppose that $|A| = m$ and $|B| = n$. Find the given cardinalities.

   (a) $|\mathcal{P}(\mathcal{P}(A))|$
   (b) $|\mathcal{P}(A \times \mathcal{P}(B))|$
   (c) $|\mathcal{P}(A) \times \mathcal{P}(B)|$
   (d) $|\{X \subseteq \mathcal{P}(A) \colon |X| \leq 1\}|$

6. You have two strings of fuse. When lit at one end, each will burn for exactly one hour. The fuses are not necessarily identical, though, and do not burn at a constant rate. All you have with you is a lighter and these two fuses. Can you measure exactly 45 minutes? If so, explain how. If not, explain why.