Directions: You may work to solve these problems in groups, but all written work must be your own. Unless the problem indicates otherwise, all problems require some justification; a correct answer without supporting reasoning is not sufficient. See "Guidelines and advice" on the course webpage for more information.

1. Express the following sets using a list between braces, using the ellipses if necessary.

    (a) $\{x \in \mathbb{N}: \ -2 < x \leq 7\}$

    (b) $\{5x: \ x \in \mathbb{Z} \text{ and } |2x| \leq 8\}$

    (c) $\{x \in \mathbb{R}: \ \cos x = 1\}$

    (d) $\{x \in \mathbb{Z}: \ |2x| < 8\}$

2. Determine whether the following sets are infinite or finite. If the set is finite, then determine its cardinality.

    (a) $\{0, \emptyset\}$

    (b) $\{1, 2, \{1, 2\}, \{1, 2, 1\}\}$

    (c) $\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$

    (d) $\{n \in \mathbb{N}: \ n \text{ is prime and } n \leq 25\}$

    (e) $\{x \in \mathbb{Q}: \ 3 \leq x \leq \pi\}$

    (f) $\{x \in \mathbb{R}: \ x^3 - x = 0\}$.

    (g) $\{A: \ A \text{ is a set with } |A| = 0\}$.

3. Use set-builder notation to express the following sets in a compact way.

    (a) $\{5, 6, 7, 8, 9, 10\}$

    (b) $\{0, 4, 16, 36, 64, 100, \ldots\}$

    (c) $\{3, 6, 11, 18, 27, 38, \ldots\}$

    (d) The set of points in the plane $\mathbb{R}^2$ that are contained inside the boundary of the square centered at $(0, 0)$ with side length 1.

    (e) The set of points in the plane $\mathbb{R}^2$ that are contained inside or along the boundary of the square centered at $(0, 0)$ with side length 1.

4. Sketch the following sets of points in the x, y-plane $\mathbb{R}^2$. Use dashes to denote boundaries that are excluded from the set.

    (a) $\{(x, y) \in \mathbb{R}^2: \ x > 1\}$.

    (b) $\{(x, x + y): \ x \in \mathbb{R} \text{ and } y \in \mathbb{Z}\}$

5. A fly is resting on the front of a train that is hurtling forward at 60 kilometers per hour. On the same track, 300 kilometers straight ahead, another train is hurtling towards the first train at 60 kilometers per hour. At that moment, when the trains are 300 km apart, the fly takes off at 90 km per hour. He continually flies back and forth between the trains, flying just above the track and instantaneously turning around when he reaches a train. What is the total distance traveled by the fly before the two trains crash together, squishing the fly between them in the process?