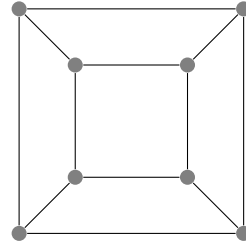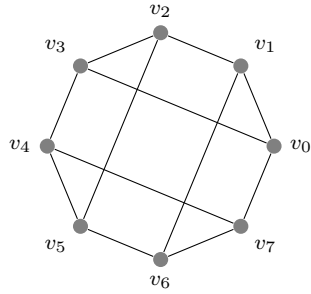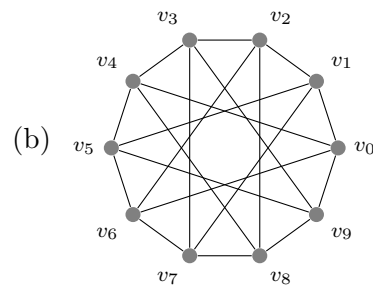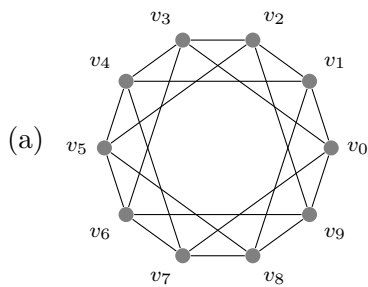Name: _____

**Directions:** Show all work. No credit for answers without work.

1. [**15 points**] Are the following graphs isomorphic? Either give an isomorphism or explain why not.

2. [**2 parts, 5 points each**] For each of the following graphs, decide whether the graph is bipartite or not. If bipartite, then give a bipartition; otherwise, explain why the graph is not bipartite.

3. [**10 points**] Let G_n be the graph whose vertices are the lattice points of an $n \times n$ grid, where two points are adjacent if and only if they are in the same row or in the same column. A copy of G_3 appears below.
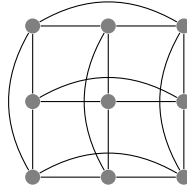
Find a formula for the number of edges in G_n.

4. [**15 points**] Recall that $K_{1,3}$ is the complete bipartite graph with one singleton part and another part with 3 vertices. Prove that $r(K_{1,3}, C_4) = 6$. [Hint: use that $r(C_4, C_4) = 6$.]

5. [**3 parts, 10 points each**] How many numbers in $\{1, \ldots, 700\}$:

   (a) have only even digits?

   (b) have 2 odd digits and 1 even digit?

   (c) have no repeated digits?

6. Suppose we roll a 6-sided die n times. Let A_n be the event that we do not roll a pair of 6's twice in a row.

   (a) [**15 points**] Let $f(n) = |A_n|$. Find a recurrence relation for $f(n)$. Include necessary base cases.

   (b) [**5 points**] Use your recurrence in part (a) to find the probability of not getting 6's twice in a row when rolling a die 4 times.