Name: Solutions

Directions: Show all work.

1. **[3 points]** A store sells packs of light bulbs in colors red, blue, green, purple, orange, and white. How many ways are there to purchase 8 packs of light bulbs?

6 types of packs: $x_1 + \cdots + x_6 = 8$

8 stars, 5 bars

$\Rightarrow \binom{8+5}{5} = \boxed{\binom{13}{5}} = 1287$ ways

2. **[4 points]** How many integer solutions are there to $x_1 + x_2 + x_3 = 50$ such that $x_1 \geq 4$, $x_2 \geq -8$, and $x_3 \geq 0$?

$\hat{x}_1 = x_1 - 4$

$\hat{x}_2 = x_2 + 8$

$\hat{x}_3 = x_3$

$x_1 + x_2 + x_3 = 50$

$(\hat{x}_1 + 4) + (\hat{x}_2 - 8) + \hat{x}_3 = 50$ $\quad \hat{x}_1, \hat{x}_2, \hat{x}_3 \geq 0$

$\hat{x}_1 + \hat{x}_2 + \hat{x}_3 = 54$

54 stars, 2 bars $\Rightarrow \boxed{\binom{56}{2}}$ solns

$= 1540$

3. **[3 points]** How many ways are there to choose 3 integers from $\{1, \ldots, 20\}$ if every chosen integer must be at most distance 9 from some other chosen integer? For example, $\{3, 12, 17\}$ works since $|12 - 3| \leq 9$ and $|17 - 12| \leq 9$, but $\{3, 13, 17\}$ does not since $|13 - 3| = 10 > 9$.

1 (2) 3 4 5 (6) 7 ... 17 (18) 19 20

x_1 x_2 x_3 x_4

Let x_1, x_2, x_3, x_4 describe the number of integers before/after chosen integers.

We want to count # solns to

$x_1 + x_2 + x_3 + x_4 = 17, \quad x_2, x_3 \leq 8$

U: all solns to $x_1 + \cdots + x_4 = 17$:
17 stars, 3 bars $\Rightarrow \binom{20}{3}$

solns w $x_2 \geq 9$: $\hat{x}_1 + \hat{x}_2 + \cdots + \hat{x}_4 = 8$ } 8 stars, 3 bars $\Rightarrow \binom{11}{3}$

Also, # solns w $x_3 \geq 9$: $\binom{11}{3}$.

No soln has both x_2 and $x_3 \geq 9$. So total # good solns:

$\boxed{\binom{20}{3} - 2\binom{11}{3}} = 810$