

Name: Solutions

Directions: Show all work.

1. [3 points] A store sells packs of light bulbs in colors red, blue, green, purple, orange, and white. How many ways are there to purchase 8 packs of light bulbs?

6 types of packs: $x_1 + \dots + x_6 = 8$
8 stars, 5 bars

$$\Rightarrow \binom{8+5}{5} = \boxed{\binom{13}{5}} = 1287 \text{ ways}$$

2. [4 points] How many integer solutions are there to $x_1 + x_2 + x_3 = 50$ such that $x_1 \geq 4$, $x_2 \geq -8$, and $x_3 \geq 0$?

$$\hat{x}_1 = x_1 - 4$$

$$\hat{x}_2 = x_2 + 8$$

$$\hat{x}_3 = x_3$$

$$x_1 + x_2 + x_3 = 50$$

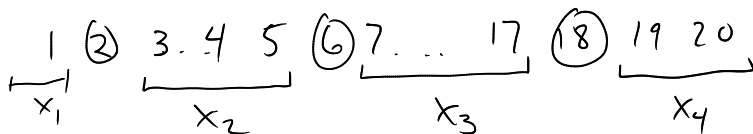
$$(\hat{x}_1 + 4) + (\hat{x}_2 - 8) + \hat{x}_3 = 50 \quad \hat{x}_1, \hat{x}_2, \hat{x}_3 \geq 0$$

$$\hat{x}_1 + \hat{x}_2 + \hat{x}_3 = 54$$

$$54 \text{ stars, } 2 \text{ bars} \Rightarrow \boxed{\binom{56}{2}} \text{ solns}$$

$$= 1540$$

3. [3 points] How many ways are there to choose 3 integers from $\{1, \dots, 20\}$ if every chosen integer must be at most distance 9 from some other chosen integer? For example, $\{3, 12, 17\}$ works since $|12 - 3| \leq 9$ and $|17 - 12| \leq 9$, but $\{3, 13, 17\}$ does not since $|13 - 3| = 10 > 9$.



Let x_1, x_2, x_3, x_4 describe the number of integers before/after chosen integers.

We want to count # solns to

$$x_1 + x_2 + x_3 + x_4 = 17, \quad x_2, x_3 \leq 8$$

$$U: \text{ all solns to } x_1 + \dots + x_4 = 17; \quad \left. \begin{array}{l} \text{solns w } x_2 \geq 9: \\ \hat{x}_1 + \hat{x}_2 + \dots + \hat{x}_4 = 8 \end{array} \right\} \begin{array}{l} 8 \text{ stars, } \\ 3 \text{ bars} \end{array} \Rightarrow \binom{11}{3}$$

Also, # solns w $x_3 \geq 9$:
 $\binom{11}{3}$.

No soln has both x_2 and $x_3 \geq 9$. So total # good solns!

$$\boxed{\binom{20}{3} - 2\binom{11}{3}} = 80$$