Name: Solutions

Directions: Show all work.

1. **[3 points]** How many ways are there to shuffle a standard 52-card playing deck so that all spades are consecutive?

1. Choose order for the 13 spades: $13!$ options

2. Arrange the 39-non spades $39!$ options

3. In one of the 40 available spaces, insert the spades 40 options

$\underbrace{\uparrow c_1 \uparrow c_2 \cdots \uparrow c_{39} \uparrow}_{40 \text{ insertion points}}$

Total: $13! \cdot 39! \cdot 40$ or $\boxed{40! \cdot 13!}$

2. **[3 points]** How many (8×8)-matrices are there such that each entry is a zero or a one, and each row has exactly 3 ones?

- In each row, we have $\binom{8}{3}$ options
- For $1 \le i \le 8$, in stage i, we choose 3 positions to have value one $\binom{8}{3}$ options, each stage

Since we have 8 stages, the total num. is $\boxed{\binom{8}{3}^8}$.

3. **[4 points]** Suppose ~~$n \geq 2$~~ $n \geq 3$. How many circular arrangements of $\{1, \ldots, n\}$ are there if 1 and n are not allowed to be consecutive? If a circular arrangement is chosen at random, what is the probability that 1 and n are not consecutive?

Let $\mathcal{U}$ be the set of all circular arrangements; we know $|\mathcal{U}| = \frac{n!}{n} = (n-1)!$

Let A be the set of arrangements in $\mathcal{U}$ with 1 and n not consecutive,

so that $\overline{A}$ is the set of circular arrangements with 1 and n consecutive.

n: 6

5 •

2 • • 3

• 4

i 1

To count $\overline{A}$, first give a circular arrangement of $\{2, \ldots, n-1\}$ ($(n-3)!$ options)

and then insert $[1\ n]$ or $[n\ 1]$ in one of the $n-2$ gaps ($2(n-2)$ options).

So $|\overline{A}| = (n-3)! \cdot 2(n-2) = 2(n-2)!$ and $|A| = |\mathcal{U}| - |\overline{A}| = (n-1)! - 2(n-2)! = (n-1)(n-2)! - 2(n-2)!$

$= \boxed{(n-3)(n-2)!}$. The probability is $\frac{(n-3)(n-2)!}{(n-1)!} = \boxed{\frac{n-3}{n-1}} = \boxed{1 - \frac{2}{n-1}}$.