Name: Solutions

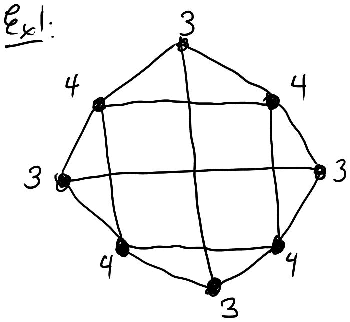
Directions: Show all work.

1. **[2 parts, 3 points each]** Graphs with specified degrees.

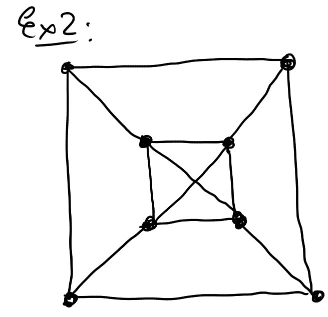
   (a) Is there an 8-vertex graph in which half the vertices have degree 3 and the other half have degree 4? Either give an example or explain why not.

Yes, it is possible. Many examples exist

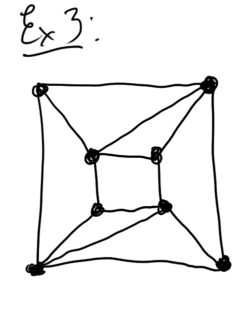
Ex1:



Ex2:



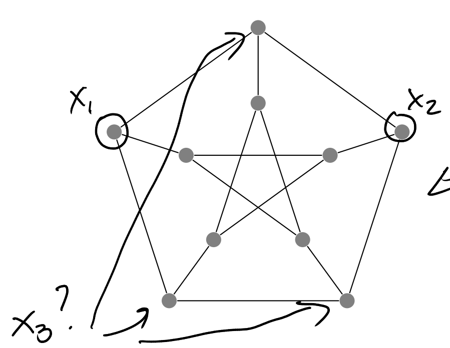
Ex3:



   (b) Is there a 10-vertex graph in which half the vertices have degree 3 and the other half have degree 4? Either give an example or explain why not.

$\boxed{\text{No, this is not possible.}}$ With 5 vertices of degree 3 and 5 vertices of degree 4, the number of vertices of odd degree is odd, which is impossible. Alternatively, such a graph would have degree sum $\sum_v d(v) = 5\cdot 3 + 5\cdot 4 = 37$. Since $\sum_v d(v) = 2|E(G)|$ by the handshaking lemma, the degree sum is always even, so no such graph exists.

2. **[4 points]** Use the Petersen graph (displayed below) to show that $K_{10} \not\to K_3, K_5$. What can we conclude about $r(3,5)$?



blue edges

Color K_{10} so that the blue subgraph is the Petersen graph, as shown above. Color the remaining pairs of vertices red. The Petersen consists of two disjoint 5-cycles joined by a matching, and therefore has no K_3. So our coloring has no blue K_3. A red K_5 is also impossible: either the inner or outer blue 5-cycle would have at least 3 vertices in the red K_5, but some pair would then be consecutive on the blue 5-cycle, giving a blue edge. We conclude $r(3,5) > 10$ and so $\boxed{r(3,5) \geq 11}$.