Name: Solutions

Directions: Show all work.

1. **[2 parts, 4 points each]** Lists and monotone subsequences.

   (a) Give a list of 15 distinct integers that has no increasing subsequence of size 4 and no decreasing subsequence of size 6.

   $\boxed{5, 4, 3, 2, 1, \ 10, 9, 8, 7, 6, \ 15, 14, 13, 12, 11}$ or

   $\boxed{13, 14, 15, \ 10, 11, 12, \ 7, 8, 9, \ 4, 5, 6, \ 1, 2, 3}$

   (b) Prove that every list of 16 distinct integers either contains an increasing subequence of size 4 or a decreasing subsequence of size 6.

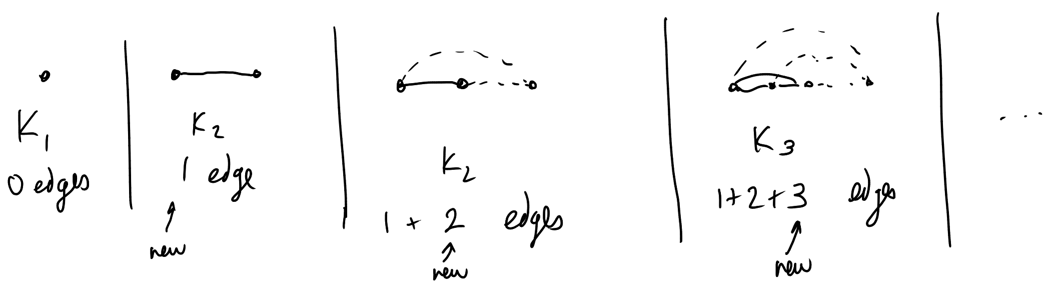
   Let $a_1, \dots, a_{16}$ be a list of distinct integers. For $1 \le i \le 16$, let

   r_i = max size of an increasing subsequence ending at a_i

   s_i = max size of a decreasing subsequence ending at a_i.

   Note that $r_i \ge 1$ and $s_i \ge 1$ for each i. Suppose for a contradiction that $r_i \le 3$ and $s_i \le 5$ for each i. Since $(r_1, s_1), \dots, (r_{16}, s_{16})$ is a list of 16 ordered pairs, each of which belongs to $\{1,2,3\} \times \{1,2,3,4,5\}$ (a set of size 15), by the pigeonhole principle there exists i and j with $1 \le i < j \le 16$ such that $(r_i, s_i) = (r_j, s_j)$. But this is impossible: if $a_i < a_j$, then $r_j \ge r_i + 1$ and if $a_i > a_j$, then $s_j \ge s_i + 1$. Therefore some r_i is at least 4 or some s_i is at least 6. ∎

2. **[2 points]** How many edges are in K_6, the complete graph on 6 vertices?



So K_6 has $1+2+3+4+5$ or $\boxed{15}$ edges.

Alternative Soln:

$$\sum_{v \in V(K_6)} d(v) = 2|E(K_6)|$$

$$6 \cdot 5 = 2|E(K_6)|$$

$$|E(K_6)| = \frac{6 \cdot 5}{2} = \boxed{15}$$