Directions: Solve the following problems. All written work must be your own. See the course syllabus for detailed rules.

1. [2.3.14] In class, we showed that $r(3,3,3) \leq 17$. What upper bound do our techniques give on $r(3,3,3,3)$?

2. Find $r(C_3, C_4)$.

3. Here, we show that $r(P_4, K_n) = 3(n-1)+1$.

   (a) With $s = 3(n-1)$, find a blue/red coloring of the edges of K_s that avoids a blue copy of P_4 and a red copy of K_n.

   (b) Prove that if G is a graph in which every vertex has degree at least 3, then G contains P_4 as a subgraph.

   (c) With $s = 3(n-1)+1$, prove that every blue/red coloring of the edges of K_s contains a blue copy of P_4 or a red copy of K_n. (Hint: if every vertex in G has at least 3 blue neighbors, then apply part (b) to the blue subgraph. Otherwise, G has a vertex u with blue degree at most 2. How large is the red neighborhood of u?)

   Comment: the argument above can be generalizes to longer paths (and even to other graphs called trees). Do you see how it generalizes to give $r(P_5, K_n)$?