Directions: Solve the following problems. All written work must be your own. See the course syllabus for detailed rules.

1. Let $n \geq 1$, let O_n be the set of odd numbers in $[n]$, and let E_n be the set of even numbers in $[n]$. Give a combinatorial proof that $\sum_{k \in E_n} \binom{n}{k} = \sum_{k \in O_n} \binom{n}{k}$.

2. [5.2.9] Let $n \geq 1$, let O_n be the set of odd numbers in $[n]$, and let E_n be the set of even numbers in $[n]$. Let $a_n = \sum_{k \in E_n} k\binom{n}{k}$ and $b_n = \sum_{k \in O_n} k\binom{n}{k}$.

   (a) Use the binomial theorem to find expressions for $a_n + b_n$ and $a_n - b_n$.

   (b) Find formulas for a_n and b_n.

3. [5.2.17] For which pairs of matrices A and B is it the case that the matrix analogue $(A+B)^n = \sum_{k=0}^{n} \binom{n}{k} A^k B^{n-k}$ of the binomial theorem holds?

4. [8.1.3] How many permutations of the letters in SCRIPPS have no two consecutive letters the same?

5. How many permutations of the letters in AABBCC...ZZ have no two consecutive letters the same? Find a summation formula.