Name: _____

**Directions:** Show all work. No credit for answers without work.

1. Recall the affine cipher with $\mathcal{K} = \mathbb{F}_p^* \times \mathbb{F}_p$, $\mathcal{M} = \mathcal{C} = \mathbb{F}_p$, encryption function $e_k(m) = \alpha m + \beta$, and decryption function $d_k(c) = \alpha^{-1}(c - \beta)$, where the key $k$ is the pair $(\alpha, \beta)$.

   (a) **[3 points]** Alice and Bob choose $p = 149$ and key $k = (43, 16)$. Alice wishes to send Bob the message $m = 101$. What ciphertext should she send?

   (b) **[4 points]** Alice receives the ciphertext $c = 20$ from Bob. What is the corresponding plaintext message?

2. **[3 points]** Alice and Bob still use the affine cipher with $p = 149$ but start using a new key $(\alpha, \beta)$. Eve obtains two plaintext/ciphertext pairs: $(m_1, c_1) = (32, 81)$ and $(m_2, c_2) = (33, 123)$. Help Eve obtain the secret key $(\alpha, \beta)$.