Directions: Solve the following problems. All written work must be your own. See the course syllabus for detailed rules.

1. Solve the following systems of congruences.

   (a)

$$\begin{aligned} x &\equiv 18 \pmod{25} \\ x &\equiv 7 \pmod{11} \\ x &\equiv 16 \pmod{32} \end{aligned}$$

   (b)

$$\begin{aligned} 17x &\equiv 8 \pmod{43} \\ 6x &\equiv 41 \pmod{55} \\ 5x &\equiv 4 \pmod{9} \end{aligned}$$

   (c)

$$\begin{aligned} 7x &\equiv 33 \pmod{145} \\ 11x &\equiv 44 \pmod{45} \\ 17x &\equiv 38 \pmod{75} \end{aligned}$$

   Note: The given moduli are not pairwise relatively prime (for example, $3 \mid 45$ and $3 \mid 75$), so CRT does not apply directly.

2. Alice and Bob wish to use the ElGamal cryptosystem to communicate, and they are having difficulty deciding on a prime/base pair (p, g). The pairs that they are considering are $(345601, 71482)$ (option A), $(516163, 482305)$ (option B), and $(177007, 145014)$ (option C). Which option do you recommend for Alice and Bob, and why?