Directions: Solve the following problems. All written work must be your own. See the course syllabus for detailed rules.

1. [JJJ 1.28] Compute the following values of the order function.
   (a) $\text{ord}_2(2816)$
   (b) $\text{ord}_7(2222574487)$
   (c) $\text{ord}_p(46375)$ for each prime $p \in \{3, 5, 7, 11\}$.

2. [JJJ 1.29] Let p be a prime number, and let a and b be positive integers. Prove the following.
   (a) $\text{ord}_p(ab) = \text{ord}_p(a) + \text{ord}_p(b)$
   (b) $\text{ord}_p(a + b) \geq \min\{\text{ord}_p(a), \text{ord}_p(b)\}$
   (c) If $\text{ord}_p(a) \neq \text{ord}_p(b)$, then $\text{ord}_p(a + b) = \min\{\text{ord}_p(a), \text{ord}_p(b)\}$.

3. Modular exponentiation in $\mathbb{F}_7$.
   (a) Fill in the table so that row a and column k contains a^k, where $a^k \in \mathbb{F}_7$.

a^k	0	1	2	3	4	5	6	7	$\cdots$
0	1	0	0	0	0	0	0	0	
1									
2									
3									
4									
5									
6									

   (b) For each non-zero element a, find the order of a in $\mathbb{F}_7^*$.
   (c) Use the table to find all primitive roots in $\mathbb{F}_7$. Verify that the number of primitive roots equals $\phi(6)$.

4. Use Fermat's Little Theorem and the fast power algorithm to compute the multiplicative inverse of 68 in $\mathbb{F}_{101}$.