Directions: Solve the following problems. All written work must be your own. See the course syllabus for detailed rules.

1. Alice and Bob wish to share a secret using Elliptic Curve-based Diffie–Hellman. They agree on the curve E given by $y^2 = x^3 + 14x + 2$ over $\mathbb{F}_{31}$ and the base element $g = (12, 10)$.
   (a) Bob picks $b = 10$ as his private exponent. What should Bob send to Alice?
   (b) Alice sends $A = (18, 17)$ to Bob. Compute Alice and Bob's shared secret.
   (c) **[Challenge (optional)]** Find Alice's private exponent a. In other words, find a such that $g^a = A$. This is an instance of the Elliptic Curve Discrete Logarithm Problem (ECDLP).

2. Given polynomials $\mathbf{a}$ and $\mathbf{b}$ in $\mathbb{F}_{11}[x]$, find $q, r \in \mathbb{F}_{11}[x]$ such that $\mathbf{a} = \mathbf{q} \cdot \mathbf{b} + \mathbf{r}$ and either $\mathbf{r} = 0$ or $\deg(\mathbf{r}) < \deg(\mathbf{b})$.
   (a) $\mathbf{a} = x^2 + 2x + 1$, $\mathbf{b} = x^3$
   (b) $\mathbf{a} = x^3$, $\mathbf{b} = x^2 + 2x + 1$
   (c) $\mathbf{a} = 3x^4 - 7x + 1$, $\mathbf{b} = x^3 + 5x^2 - 4$
   (d) $\mathbf{a} = x^3 - x^2 - 3x + 1$, $\mathbf{b} = 7x^2 + 4x - 3$